Recommended by the Ministry of Education of the Republic of Kazakhstan

Akzhol Yelemessov
Yernar Bazarov
Azat Turapbekov

ALGEBRA

Grade 10

Textbook

ASTANAKITAP
Almaty 2023

UDC 373.167.1
LBC 22.14 я72

Yelemessov A.
Y40 **ALGEBRA, Grade 10: Textbook**/ Yernar Bazarov,
Akzhol Yelemessov, Azat Turapbekov
- Almaty: Астана-кітап, 2023 - 246 p.
ISBN 978-601-80993-6-6

UDC 373.167.1
LBC 22.14 я72

ISBN 978-601-80993-6-6

PREFACE

This book follows the Algebra Syllabus for 10th form, implemented from 2017 by the Ministry of Education of the Republic of Kazakhstan.

Mathematics has many branches and makes it easier to solve a wide variety of problems. The goal of this text is to help students develop the skills necessary for solving algebraic problems, and then help students apply these skills. By the end of the book, students will have a good understanding of the analytic approach to solving problems. In addition, we have provided many systematic explanations throughout the text that will help instructors to reach the goals that they have set for their students. As always, we have taken particular care to create a book that students can read, understand, and enjoy, and that will help students gain confidence in their ability to use mathematics.

This book consists of nine chapters, which cover PROPERTIES AND GRAPHS OF FUNCTIONS, TRIGONOMETRIC FUNCTIONS, TRIGONOMETRIC EQUATIONS AND INEQUALITIES, PROBABILITY, POLYNOMIALS, LIMITS, DERIVATIVES, APPLICATION OF DERIVATIVES, RANDOM VARIABLES, respectively. Each chapter begins with basic definitions, theorems, and explanations which are necessary for understanding the subsequent chapter material. In addition, each chapter is divided into subsections so that students can follow the material easily.

The language of this book is more student-friendly rather than purely mathematical. Deep mathematical notation is especially avoided in order to not lead to confusion. The book tries to explain the topic as a teacher would explain it in the classroom, giving examples and exercises which prompt the student to think for him or herself.

CONTENTS

CONTENTS

CONTENTS

CONTENTS

CONTENTS

CHAPTER 9: RANDOM VARIABLES

1

CHAPTER

FUNCTIONS, ITS PROPERTIES AND GRAPHS

As we know, a function is a machine that takes an input and returns an output by applying a specific rule. For example, the foundation of every state is education of its youth (Diogenes), where input is youth, function stands for education and output is the great state.

1.1 UNDERSTANDING FUNCTION AND ITS GRAPH

You will

- *define the function, and terms like "domain" and "range";*
- *understand function notation;*
- *be able to do transformations on graphs of a function.*

a. Function Notation

You are already familiar with linear and quadratic functions and their graphs from the previous years. In this chapter we are going to investigate the properties of the graphs representing functions of various kinds. Firstly, let us recall basic concepts of a function and its notations.

Definition

A **function** f is a rule that assigns each element x in set D to exactly one element y or $f(x)$ in set E. Set D is called the **domain** and set E is called the **range** of the function f. We name x as the **independent variable** or the **argument**, and y as the **dependent variable** since the value of y depends on x.

Consider the function $f(x) = x^2$. We can write this function in different ways:

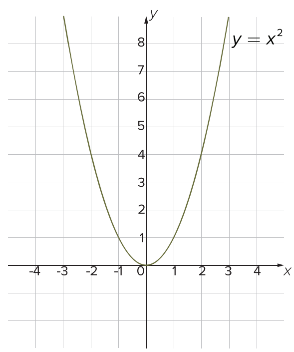
$f(x) = x^2$, $y = x^2$ or $f : x \to x^2$.

Function can be defined by *algebraic formula*, *graph* or *table*.

For example, on the left side, function $y = x^2$ is given as a formula, table and graph.

Keep in mind

1. According to the definition of function, there can't be two different values of a function for any argument.
2. To check whether graph belongs to a function or not, it is enough to do a vertical line test. If vertical line intersects the graph at two points, graph does not belong to a function.

Algebraic formula	Table	Graph
$y = x^2$	x: -5, -2, -1, 0, 1/2, 2, 3, 5 y: 25, 4, 1, 0, 1/4, 4, 9, 25	

b. Domain and Range of a Function

Domain is the set of all possible values of the independent variable for which function is defined. Unless stated otherwise, domain is the largest set of all real x-values. Domain is usually denoted by letter D, and $D(f)$ means domain of a function f. **Range** of a function is the set of all images of all elements in its domain. We will denote range with letter E, and $E(f)$ will mean range of a function f.

Let us look at the domain and the range of some common types of functions.

Type of function	Form	Domain	Examples
Polynomial functions	$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_0$ $n \in \mathbb{Z} - \mathbb{Z}^-$	$\mathbb{R}$	$g(x) = 4x - 3$, $D(g) = \mathbb{R}$ $h(x) = 2x^5 - 3x^4 + 5x - 1$, $D(h) = \mathbb{R}$

As written above, domain of any polynomial function is set of real numbers. The range of the polynomial function depends on the function itself. For example, as we can see from the graph of $f(x)=x^n$, range of the function depends on exponent n (see fig. 1)

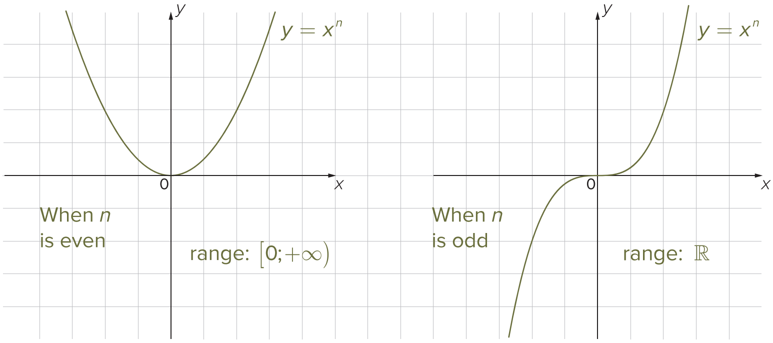


fig. 1

Type of function	Form	Domain	Examples
Rational functions	$f(x)=\dfrac{g(x)}{h(x)}$	$h(x)\neq 0$	$g(x)=\dfrac{x+1}{x-2}$, $D(g)=\mathbb{R}-\{2\}$ $h(x)=\dfrac{x+5}{x^2-x-6}$, $D(f)=\mathbb{R}-\{-2;3\}$

We know that denominator cannot be zero, so any numbers which make the denominator zero must be excluded from the domain of a rational function. As an example, we can see the fig. 2 with the graph of the function $y=x^{-n}=\dfrac{1}{x^n}$.

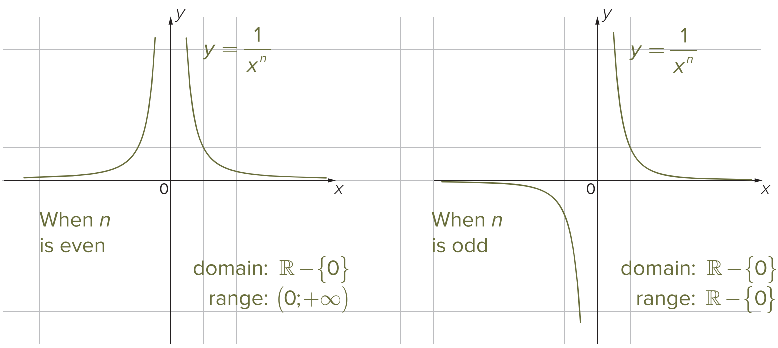


fig. 2

We can see that range of a rational function changes according to the function.

Terminology

Notation - белгіленуі / обозначение

Domain - анықталу облысы / область определения

Range - мәндер жиыны / область значения

Variable - айнымалы / переменная

Argument - аргумент

Type of function	Form	Domain	Examples
Radical functions	$f(x)=\sqrt{g(x)}$	$\{x \mid x\in\mathbb{R} \text{ and } g(x)\geq 0\}$	$f(x)=\sqrt{x^2-4}$, $D(f)=\mathbb{R}-(-2,2)$

The expression under the square root must be nonnegative, so we must exclude any numbers which make the expression negative from the domain.

Example 1

Complete the table below according to the given functions.

Function	Domain	Range
$f(x)=2x-3$		
$f(x)=x^2+4x-5$		
$f(x)=\dfrac{5}{x+2}$		
$f(x)=\dfrac{x^2+4x+3}{x+1}$		
$f(x)=\sqrt{x^2-4}$		

Left as an exercise for students.

Similar questions

Page 35, questions 1(a)-(l)

Example 2

Find the domain and range of the function $f(x)=\sqrt{x^2-6x-7}$.

Solution:
Since expression under the square root must be nonnegative, we take

$$x^2-6x-7\geq 0$$

Let us solve the inequality $x^2-6x-7\geq 0$:

		-1		7	
x^2-6x-7	$+$	●	$-$	●	$+$

The trinomial under the root is nonnegative in the intervals $(-\infty;-1]$ and $[7;+\infty)$.

Therefore domain of the function is $(-\infty;-1]\cup[7;\infty)$.

The expression $\sqrt{x^2-6x-7}$ takes all values in the interval $[0;+\infty)$. So the range is $[0;+\infty)$.

Keep in mind

If a function consist of sum or difference of different functions, then the domain of the function is the intersection of the domains of each function.

Similar questions

Page 35-36, questions 1(m)-(n), 3, 4 and 5

Example 3

Find the domain of the function $f(x)=\dfrac{x-2}{x^2+5x+6}+\sqrt{x^2-1}$.

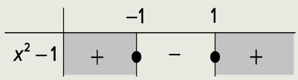
Solution:

Let us look at the function as the sum of two functions $g(x)=\dfrac{x-2}{x^2+5x+6}$ and $h(x)=\sqrt{x^2-1}$.

Denominator cannot be zero, so we take $x^2+5x+6\neq 0$.

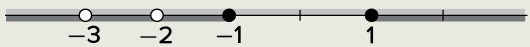
Therefore, domain of the function *g*(*x*) is $(-\infty;-3)\cup(-3;-2)\cup(-2;+\infty)$.

Expression under the square root must be nonnegative, we take $x^2-1\geq 0$.



Therefore, domain of the function $h(x)=\sqrt{x^2-1}$ is in the intervals $(-\infty;-1]$ and $[1;+\infty)$.

Domain of the function *f*(*x*) is the intersection of *D*(*g*) and *D*(*h*).



Hence, *D*(*f*) is $(-\infty;-3)\cup(-3;-2)\cup(-2;-1]\cup[1+\infty)$.

Similar questions

Page 35, questions 1(o)-(t) and 2

c. Transformation of Graphs Shift

Vertical shift	*Horizontal shift*
In general, given the graph of *y* = *f*(*x*) and *k* > 0 (fig. 3a), we can see that the graph of: 1) *y* = *f*(*x*) + *k* is obtained by shifting the graph of *y* = *f*(*x*) upward *k* units (fig. 3a); 2) *y* = *f*(*x*) – *k* is obtained by shifting the graph of *y* = *f*(*x*) downward *k* units (fig. 3a);	In general, given the graph of *y* = *f*(*x*) and *k* > 0 (fig. 3b), we can see that the graph of: 1) *y* = *f*(*x* + *k*) is obtained by shifting the graph of *y* = *f*(*x*) to the left *k* units (fig. 3b); 2) *y* = *f*(*x* – *k*) is obtained by shifting the graph of *y* = *f*(*x*) to the right *k* units (fig. 3b);

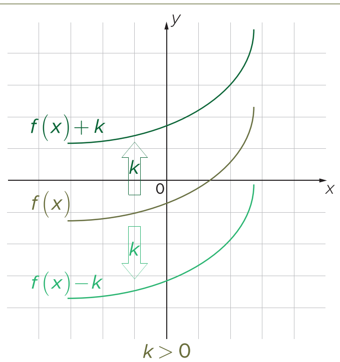


fig. 3a

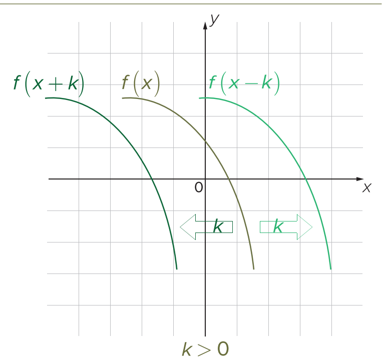


fig.3b

Terminology

Nonnegative - теріс емес / неотрицательный

Expression - өрнек / выражение

Denominator - бөлшектің бөлімі / знаменатель

Hence - демек / следовательно

Shift - жылжыту / сдвиг

Keep in mind

To sketch the graph of

$y = -f(-x + k) + p$

we apply the following steps:

1. Sketch $f(-x)$
2. Sketch $f(-(x - k))$
3. Sketch $-f(-x + k)$
4. Sketch $-f(-x + k) + p$

Example 4

Sketch the graph of the function $f(x) = (x - 2)^2 + 3$.

Solution:

We know that graph of $f(x)$ is the transformation of graph of $y = x^2$.

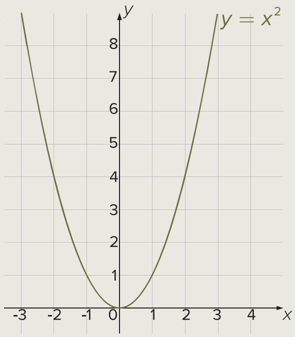
To sketch the graph of f(x) we apply the following steps:

1) Sketch the graph of $y = x^2$.

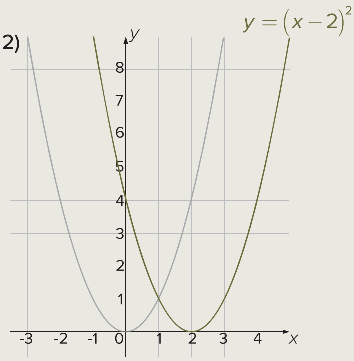
2) Sketch $y = (x - 2)^2$ by horizontal shift to the right 2 units.

3) Sketch $y = (x - 2)^2 + 3$ by vertical shift upward 3 units.

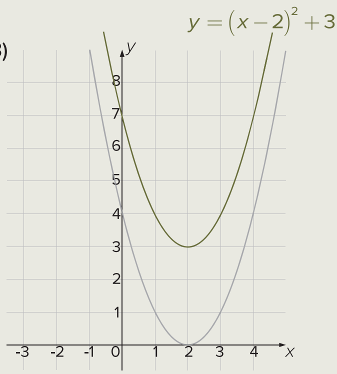
1)



2)



3)



Similar questions

Page 40, questions 16(a)-(f) and 17(a)-(d)

Reflection in x-axis	*Reflection in y-axis*
In general, given the graph of $y = f(x)$, we can see that the graph of $y = -f(x)$ is obtained by reflecting the graph of $y = f(x)$ in the x-axis, i.e (see fig. 4a).	In general, given the graph of $y = f(x)$, we can see that the graph of $y = f(-x)$ is obtained by reflecting the graph of $y = f(x)$ in the y-axis, i.e (see fig. 4b).

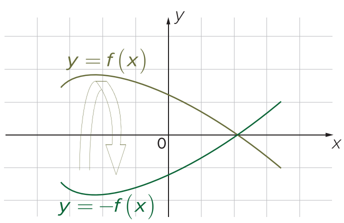


fig. 4a

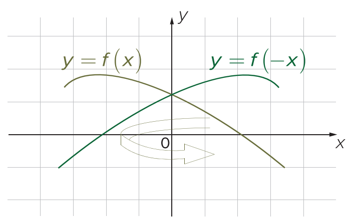


fig. 4b

Vertical Stretch and Shrink	*Horizontal Stretch and Shrink*
In general, given the graph of $y = f(x)$, we can see that the graph of (fig. 5a):	In general, given the graph of $y = f(x)$, we can see that the graph of (fig. 5b):
1) $y = k \cdot f(x)$ is obtained by stretching the graph of $y = f(x)$ vertically by a factor of k, when $k > 1$ (fig. 5a).	1) $y = f(k \cdot x)$ is obtained by shrinking the graph of $y = f(x)$ horizontally by a factor of k, when $k > 1$ (fig. 5b).
2) $y = k \cdot f(x)$ is obtained by shrinking the graph of $y = f(x)$ vertically by a factor of k, when $0 < k < 1$ (fig. 5a).	2) $y = f(k \cdot x)$ is obtained by stretching the graph of $y = f(x)$ horizontally by a factor of k, when $0 < k < 1$ (fig. 5b).

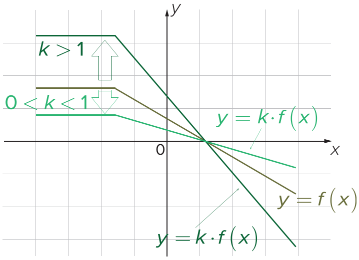


fig. 5a

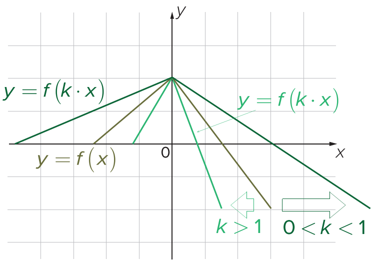


fig. 5b

Example 5

Sketch the graph of the function $f(x) = -2\sqrt{x+1} - 3$.

Solution:

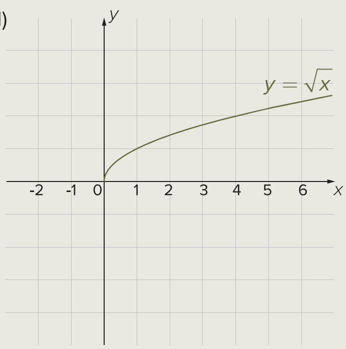
We know that graph of $f(x)$ is the transformation of graph of $y = \sqrt{x}$.

To sketch the graph of f(x) we do the next:

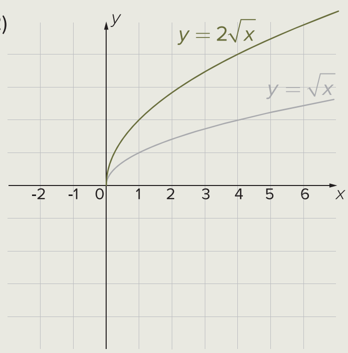
1) Sketch the graph of $y = \sqrt{x}$.

2) Sketch the graph of $y = 2\sqrt{x}$ by stretching vertically (multiply each y-coordinate by 2).

1)



2)



3) Sketch the graph of $y = -2\sqrt{x}$ by reflecting vertically the graph in the x-axis.

Research time

A graphing calculator (Desmos, GeoGebra, e.t.c.) can be used to draw the graph of a function. The graph of

$$f(x) = (x-2)^2 + 1$$

and

$$g(x) = -2\sqrt{x+1} - 2$$

are shown in fig. 6a and fig. 6b.

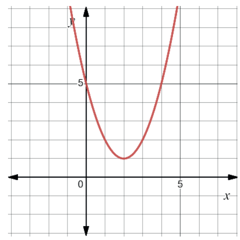


fig. 6a

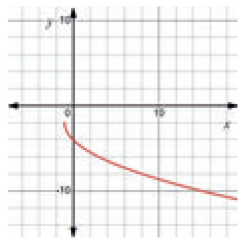


fig. 6a

Learn more about graphing calculators and try draw the graphs following functions:

- $f(x) = 6 - (x+3)^2$,
- $g(x) = \sqrt{x+4} - 2$.

Terminology

To sketch - сызу / рисовать

Reflection - шағылу / отражение

To stretch - созу / растягивать

To shrink - сығу / сжать

Keep in mind

Given the graph of $f(x)$ and $a,b,c,d \in \mathbb{R}$, to sketch the graph of $a \cdot f(bx+c)+d$ we apply the following transformation order:

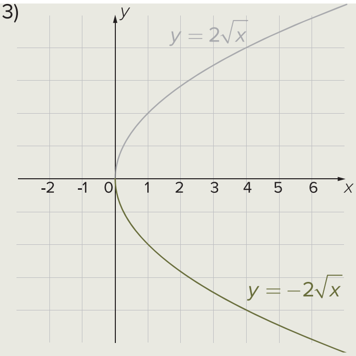
1. Sketch $f(bx)$

2. Sketch $f(b(x+\frac{c}{b}))$

3. Sketch $a \cdot f(b(x+\frac{c}{b}))$

4. Sketch $a \cdot f(b(x+\frac{c}{b}))+d$

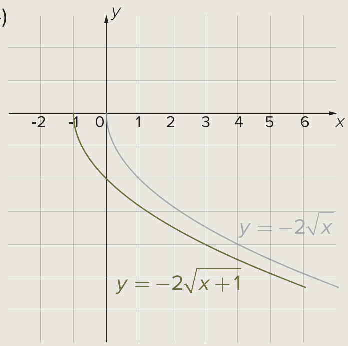
3)



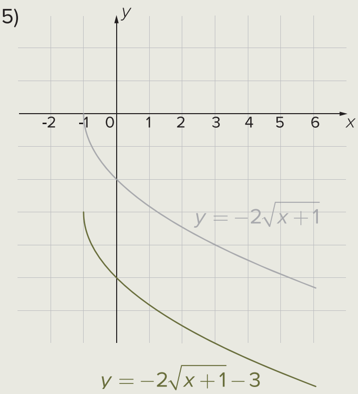
4) Sketch the graph of $y=-2\sqrt{x+1}$ by horizontal shift to the left 1 unit.

5) Sketch the graph of $y=-2\sqrt{x+1}-3$ by vertical shift downward 3 units.

4)



5)



Similar questions

Page 40, questions 14, 15, 16(g)-(l) and 17(e)-(j)

Example 6

Sketch the graph of the function $f(x)=\dfrac{x+1}{x-2}$.

Solution:

Let us write the given function as

$$f(x)=\frac{x+1}{x-2}=\frac{x-2+3}{x-2}=\frac{x-2}{x-2}+\frac{3}{x-2}=1+\frac{3}{x-2}.$$

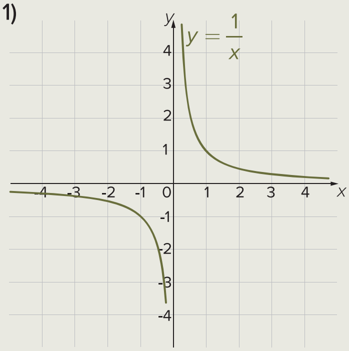
We can see that graph of $f(x)$ is the transformation of graph of $f(x)=\dfrac{1}{x}$.

To sketch the graph of f(x) we do the next:

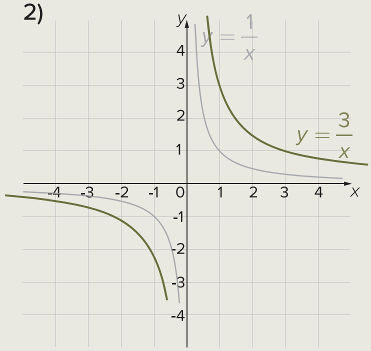
1) Sketch the graph of $y=\dfrac{1}{x}$.

2) Sketch the graph of $y=3\cdot\dfrac{1}{x}=\dfrac{3}{x}$ by stretching vertically (multiply each y-coordinate by 3).

1)



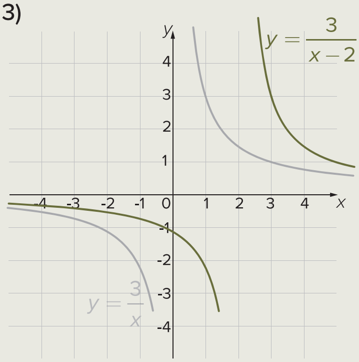
2)



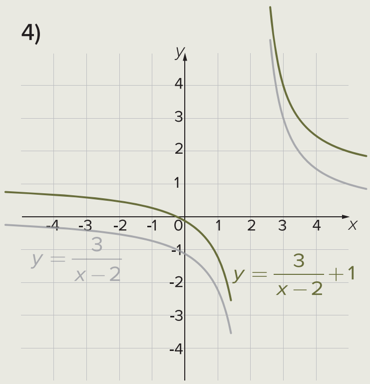
3) Sketch the graph of $y = \frac{3}{x-2}$ by horizontal shift to the right 2 units.

4) Sketch the graph of $y = \frac{3}{x-2} + 1$ by vertical shift upward 1 unit.

3)



4)



Similar questions

Page 40, questions 16(m)-(t) and 17(k)-(p)

1.2 PROPERTIES OF A FUNCTION

In this section, we will study some properties of a function. These properties will help us to read the graph of function and determine its type.

a. Zero of a Function

You will

- understand "zero of a function", "even and odd functions", "period of a function", "increasing, decreasing and constant function", "extremum of a function", "maximum and minimum value of a function", "boundedness of a function";

- be able to define properties of function according to its graph.

Definition

A value of x that makes function equal to zero is called "**zero**" (or **root**) of a function.

To find zero of a function $y = f(x)$ it is enough to solve the equation $f(x) = 0$.

For example, 2 and –3 are zeros of a function $y = x^2 + x - 6$.

On the graph (fig. 7), we can define zero(s) of a function as abscissa of the intersection point(s) with x-axis.

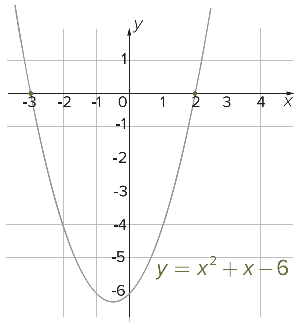


fig. 7

Example 1

Find zeros of the following functions:

a. $y = 2x - 3$ b. $y = x^2 - 3x - 18$ c. $y = \frac{x^2 - 1}{x + 2}$

Solution:

To find zero of a function we substitute 0 for y and solve the equation.

a. $y = 2x - 3$

$0 = 2x - 3$

$2x = 3$

$x = 1.5$

Hence, $x = 1.5$ is the zero of the given function.

b. $y = x^2 - 3x - 18$

$0 = x^2 - 3x - 18$

$0 = (x - 6)(x + 3)$

$x_1 = 6$ or $x_2 = -3$.

Hence, –3 and 6 are zeros of the given function.

c. $y = \frac{x^2 - 1}{x + 2}$

Similar questions

Page 39, questions 12(c)-(f) and 13(c)-(f)

$$0=\frac{x^2-1}{x+2},\ x\neq -2$$

$$x^2-1=0$$

$$x^2=1$$

$$x=\pm 1.$$

Hence, –1 and 1 are zeros of the given function.

b. Even and Odd Function

Definition

Let *f* be a function defined in D.

1. If $f(-x)=f(x)$ for all $x\in D$ then *f* is called even function.

2. If $f(-x)=-f(x)$ for all $x\in D$ then *f* is called odd function.

For instance, $f(x)=x^2$ is an even function because $f(-x)=(-x)^2=x^2=f(x)$.

Similarly, $f(x)=\frac{1}{x}$ is an odd function because $f(-x)=\frac{1}{-x}=-\frac{1}{x}=-f(x)$.

Graph of an even function

The graph of an even function is symmetric about y-axis because $(x_0; y_0)$ and $(-x_0; y_0)$ are on the graph (fig. 8a).

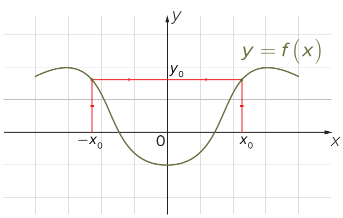


fig. 8a

Graph of an odd function

The graph of an odd function is symmetric about origin because $(x_0; y_0)$ and $(-x_0; -y_0)$ are on the graph (fig. 8b).

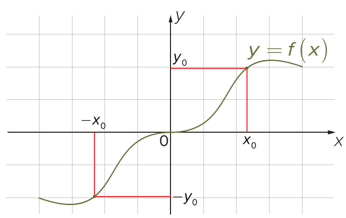


fig. 8b

If a function is not even and not odd, we say it’s “neither even nor odd”. For example, $f(x)=3x+2$ is neither even nor odd.

Example 2

Classify whether the given functions are even or odd:

a. $f(x)=-4x^2+3$ b. $f(x)=7x+5x^5$ c. $f(x)=1+\frac{2}{x}$ d. $f(x)=\frac{x^2-4}{5x}$

Solution:

Let us evaluate f(–x) and find out if it is the same function or its negative.

Keep in mind

Remember that!

From the 7th grade we know the the graph of $y=x^2$ is symmetric about the *y*-axis (see fig. 11a) and the graph of $y=x^3$ is symmetric about the origin (see fig. 11a).

That's why the quadratic function is an even function and the cube function is an odd function.

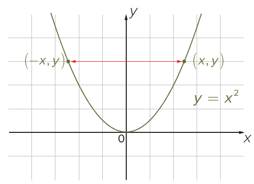


fig. 11a

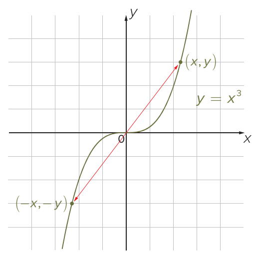


fig. 11b

Terminology

Zero of a function - функцияның нөлі / нуль функции

Even function - жұп функция / четная функция

odd function - тақ функция / нечетная функция

Similar questions

Page 36-37, questions 6, 7 and 8

Discussion

Discuss with your peers to answer the following questions and give reasons for your answer:

- Must the product of two *even* functions always be even?
- Can anything be said about the product of two *odd* functions?

a. $f(x) = -4x^2 + 3$

$f(-x) = -4(-x)^2 + 3$

$f(-x) = -4x^2 + 3$

$f(-x) = f(x).$

Function is even.

b. $f(x) = 7x + 5x^5$

$f(-x) = 7(-x) + 5(-x)^5$

$f(-x) = -7x - 5x^5$

$f(-x) = -(7x + 5x^5)$

$f(-x) = -f(x).$

Function is odd.

c. $f(x) = 1 + \dfrac{2}{x}$

$f(-x) = 1 + \dfrac{2}{-x}$

$f(-x) = 1 - \dfrac{2}{x}$

Function is neither even nor odd because $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$.

d. $f(x) = \dfrac{x^2 - 4}{5x}$

$f(-x) = \dfrac{(-x)^2 - 4}{5(-x)}$

$f(-x) = \dfrac{x^2 - 4}{-5x}$

$f(-x) = -\dfrac{x^2 - 4}{5x}$

$f(-x) = -f(x).$

Function is odd.

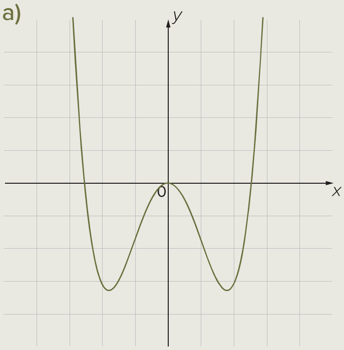
Similar questions

Page 37, questions 9 and 10

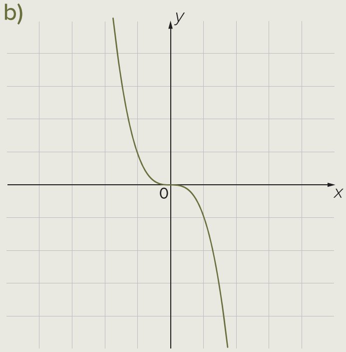
Example 3

Classify whether the given functions are even or odd:

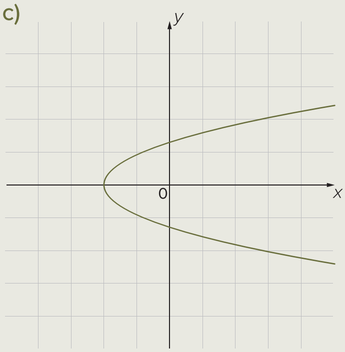
a)



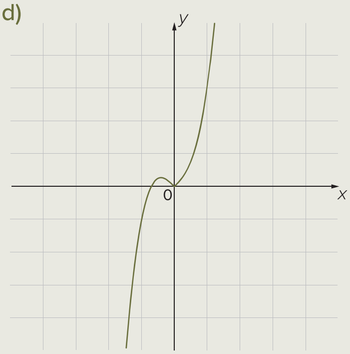
b)



c)



d)



Solution:

a. Function is an even function, since its graph is symmetric about y-axis.

b. Function is an odd function, since its graph is symmetric about origin.

c. Using the vertical line test we can define that it's not the graph of a function. Therefore we cannot talk about whether it's even or odd.

d. Function is neither even nor odd, since its graph is not symmetric.

c. Period of a Function

Definition

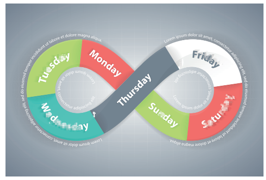
A function that repeats in regular intervals or periods is called ***periodic function***. T is said to be the ***period*** of the periodic function if it satisfies the rule $f(x+T)=f(x)$ for all values of x in its domain.

The smallest possible positive value of T is called the ***fundamental period*** of the function.

We can state that value of a periodic function repeats after the interval equal to its fundamental period. This statement can be used in sketching the graph of a periodic function.

Discussion

Think of "days of week" as a function. Discuss with your peers, can we state that the function is periodic? If so, what is its fundamental period?



Example 4

For a periodic function $f(x)$ with period $T = 7$, it's known that $f(4) = 9$. Find $f(18)$ and $f(-10)$.

Solution:

Since the period of the function is 7, we state the following:

$f(4)=9$,	$f(4)=9$,
$f(4+7)=f(11)=9$,	$f(4-7)=f(-3)=9$,
$f(11+7)=f(18)=9$.	$f(-3-7)=f(-10)=9$.

Similar questions

Page 40, questions 18 and 19

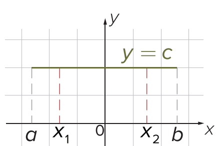
d. Constant, Increasing and Decreasing Function

Let $f(x)$ be a function defined in the interval $[a,b]$ and $x_1, x_2 \in [a,b]$ such that $x_1 < x_2$.

1. If $f(x_1)=f(x_2)$, then function $f(x)$ is called a ***constant function*** on $[a,b]$. Constant function is written as $f(x) = c$, where c is any real number (see fig. 9a).

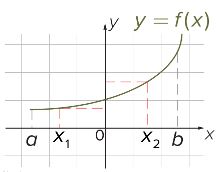
2. If $f(x_1)<f(x_2)$, then function $f(x)$ is called an ***increasing function*** on $[a,b]$ (see fig. 9b).

3. If $f(x_1)>f(x_2)$, then function $f(x)$ is called a ***decreasing function*** on $[a,b]$ (see fig. 9c).



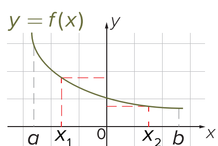
$f(x)$ is constant on $[a,b]$

fig. 9a



$f(x)$ is increasing on $[a,b]$

fig. 9b



$f(x)$ is decreasing on $[a,b]$

fig. 9c

Terminology

Period of a function - функцияның периоды / период функции

Periodic function - периодты функция / периодическая функция

Constant function - тұрақты функция / постоянная функция

Increasing function - өспелі функция / возрастающая функция

Decreasing function - кемімелі функция / убывающая функция

Example 5

Given that $f(x) = (a-3)x^2 + (b+4)x + 5$ is a constant function. Find $a - b$.

Solution:

Since $f(x)$ is a constant function, coefficients $(a - 3)$ and $(b + 4)$ must be zero. So

$a - 3 = 0$, $\quad b + 4 = 0$,

$a = 3$. $\quad b = -4$.

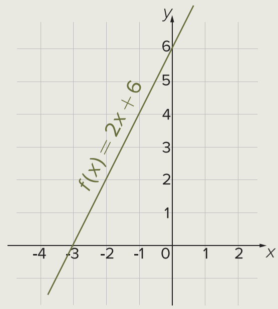
Therefore, $a - b = 3 - (-4) = 7$.

Example 6

Show that $f(x) = 2x + 6$ is increasing in $\mathbb{R}$.

Solution:

We can sketch the graph of $f(x) = 2x + 6$ to see that it is increasing in $\mathbb{R}$.



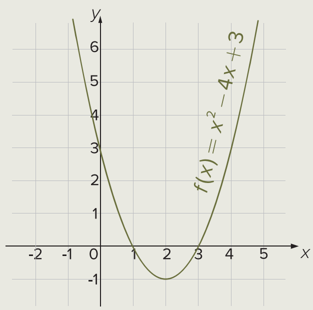
Example 7

Find intervals of increase and decrease of the function $f(x) = x^2 - 4x + 3$.

Solution:

We sketch the parabola to see the intervals of increase and decrease.

So we can see that graph of the function decreases in the interval $(-\infty; 2]$ and increases in $[2; +\infty)$.



Similar questions

Page 38-39, questions 12 and 13

e. Extremum of a Function

Definition

A function has an ***absolute maximum*** (***absolute minimum***) at point c if $f(c) > f(x)$ ($f(c) < f(x)$) for all the values of x in its domain.

In other words it's the largest (smallest) value of a function in its domain.

Definition

A function has a ***local maximum*** (***local minimum***) at point c if

$$f(c) > f(x) \ \ (f(c) < f(x))$$

for all the values of x in the given interval.

In other words it's the maximum (minimum) value of a function in a given interval.

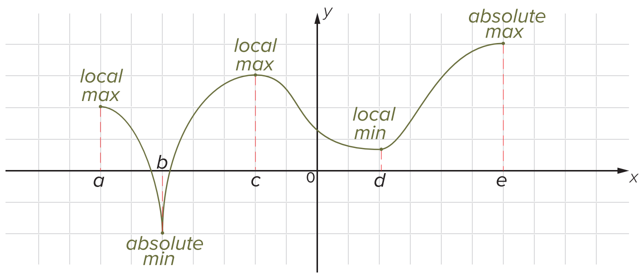
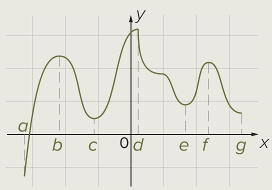


fig. 10

An ***extremum*** (plural: extrema) is either maximum or minimum value of a function. Points where function has extrema are called ***extreme points***.

Example 8

The graph of function with domain [a, g] is given. For each of the points from a to g, state whether the function has a local maximum (minimum) or an absolute maximum (minimum).



Solution:

Left as an exercise for students.

f. Boundedness of a Function

Definition

A function f which is defined in an interval, is called a **bounded function**, if the set of all outputs is bounded. We can say that if there exists a real number b in such a way that: $|f(x)| \leq b$, where x is from given interval.

In further studies we will see functions like $y = \sin x$ and $y = \cos x$ which are examples of bounded function.

Example 9

Show that $f(x) = \frac{1}{1+x^2}$ is a bounded function.

Solution:

We use $x^2 \geq 0$.

By adding 1 to both sides of inequality we get $1 + x^2 \geq 1$.

From the last inequality we obtain $\frac{1}{1+x^2} \leq 1$.

Terminology

Extremum of a function - функцияның экстремумы / экстремум функции

Absolute maximum - ең үлкен мәні / максимальное значение

Local maximum - локальді максимум / локальный максимум

Extreme point - экстремум нүктесі / точка экстремума

Boundedness of a function - функцияның шектелгендігі / ограниченность функции

Bounded function - шектелген функция / ограниченная функция