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ALGEBRA

Grade 8

Textbook

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PREFACE

Algebra 8 is designed to provide learners to improvement and development of their mathematical background needed for further algebra courses.

Starting from the fi rst pages, you will realize that this textbook is completely different from any other usual textbooks full of theoretical passages and formulas. Every chapter contains useful information, interesting facts, tasks for individual and group work. You will also learn how to conduct researches and experiments by yourselves, search for information, make your discoveries. By the end of the book, students will have a good understanding of the analytic approach to solving problems. In addition, we have provided many systematic explanations throughout the textbook that will help instructors to reach the goals that they have set for their students.

One more valuable feature of this textbook is the language. Every sentence has been carefully chosen so that it is not diffi cult for you to understand mathematics in the English language. Each page contains translations of all the important terms, both in Kazakh and Russian. This textbook will not only help you improve your English, but it will also make you a part of a big international science community.

This textbook consists of fi ve chapters, which cover SQUARE ROOTS AND IRRATIONAL EXPRESSIONS, QUADRATIC EQUATION, QUADRATIC FUNCTION, ELEMENTS OF STATISTICS, INEQUALITIES respectively. Each chapter begins with basic defi nitions, theorems, and explanations which are necessary for understanding the materials that are in the subsequent chapters. In addition, each chapter is divided into subsections so that students can follow the material easily.

Every subsection includes self-test PRACTICE problem sections. Teachers should encourage their students to solve PRACTICE problems themselves because these problems are fundamental to understanding and learning the related subjects or sections. At the end of every section, there are PROBLEMS categorized according to the structure and subject matter of the section. Problems are graded in order, from easy (A) to diffi cult (C). In addition, at the end of every chapter there is SUMMARY categorized according to the structure and subject matter of the chapter.

Please pay attention to the structure of this textbook. Remember: a textbook is no longer the only source of information in the modern world. With the help of carefully selected tasks, you are going to learn such important skills as critical thinking, problem solving, information analysis, creativity, imagination, teamwork, digital literacy, etc.

Best regards,

team of authors, "Астана-кітап"

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Chapter 1

SQUARE ROOTS AND IRRATIONAL EXPRESSIONS

1.1 REAL NUMBERS. SQUARE ROOT

1.2 TRANSFORM EXPRESSIONS CONTAINING SQUARE ROOTS

1.3 FUNCTION OF THE FORM $y = \sqrt{x}$, ITS GRAPH AND PROPERTIES

1.1 REAL NUMBERS. SQUARE ROOTS

You will:

- learn concepts of irrational and real numbers;
- know the definition of a square root;
- apply the properties of square root;
- estimate the value of the square root.

UNDERSTANDING REAL NUMBERS

The real number system evolved by expanding what we mean by the word "number". At first, "number" meant something we could count, such as the number of sheep a farmer owns.

These are called **natural numbers**.

For example, $\mathbb{N} = 1, 2, 3, 4, \ldots$

At some point, the idea of "zero" came to be considered a number. If the farmer does not have any sheep, then the number of sheep that the farmer owns is zero. So we call the set of natural numbers plus the number zero the **whole numbers**.

The set of whole numbers is the set of natural numbers and zero. It is denoted by *W*.

For example, $W = 0, 1, 2, 3, 4, \ldots$

Note

A **decimal** is a fraction written in a special form. Instead of writing 1/2, for example, you can express the fraction as the **decimal** 0.5.

The set of **integers** is the set of natural numbers, zero and the negatives of the natural numbers. It is denoted by $\mathbb{Z}$.

For example, $\mathbb{Z} = \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$

Rational and Irrational Numbers

Recall that a simple fraction is a fraction with integers in its numerator and denominator.

For example, $\frac{4}{1}, \frac{3}{2}, \frac{1}{3}, -\frac{237}{125}$, and $\frac{-37}{1}$.

A **rational number** is a number that can be expressed as a simple fraction, denoted by $\mathbb{Q}$.

The ancient Greeks was the first to discover that there are numbers that are not rational numbers. They called them **irrational**. An irrational number is not a rational number. The set of irrational numbers is denoted by $\mathbb{Q}'$. And any irrational numbers have the following properties:

- It cannot be expressed as a fraction.
- As decimals, they never repeat or terminate.

Fact

π

π (Pi) is a famous irrational number. We cannot write down a simple fraction that equals π. The popular approximation of 3.14 is close but not accurate. Pi, in mathematics, the ratio of the circumference of a circle to its diameter.

For examples,

$\frac{3}{2} = 1.5$ Rational (terminate)

$\frac{1}{3} = 0.333\overline{3}$ Rational (repeats)

$\sqrt{2} = 1.414213...$ Irrational (never repeats and terminate)

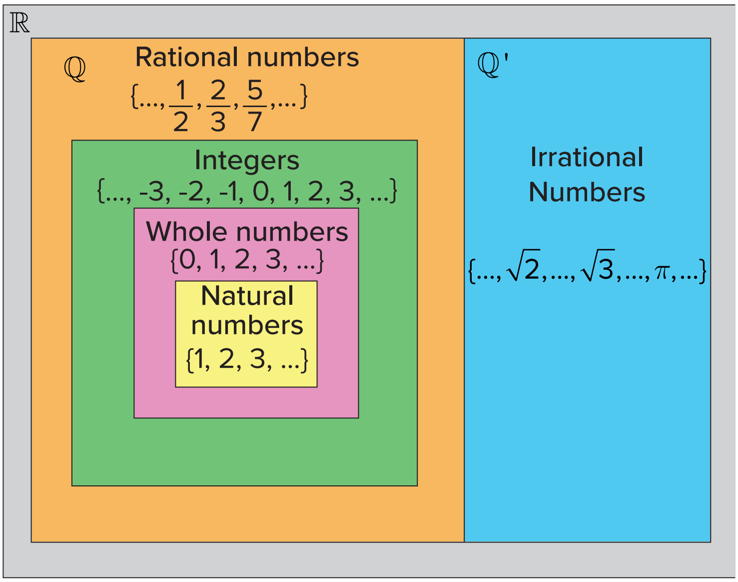
$\pi = 3.141592...$ Irrational (never repeats and terminate)

Real Numbers

The union of the set of rational and irrational numbers forms the set of **real numbers**, and the set of real numbers is denoted by $\mathbb{R}$.

When we combine irrational numbers with rational numbers, we finally have the complete set of real numbers. Any number representing an amount of something, such as a weight, a volume, or the distance between two points, will always be a real number. The following diagram illustrates the relationships of the sets that make up the real numbers.

Real Numbers



$\mathbb{N} \in W \in \mathbb{Z} \in \mathbb{Q} \in \mathbb{R}$

$\mathbb{Q}' \in \mathbb{R}$

Activity

How do we know that certain numbers are irrational?

If we evaluate $\sqrt{2}$ on a calculator, you will see a decimal approximation. One calculator shows 1.414213562. Another shows 1.41421356237. No matter how many decimal places the calculator shows, it is not enough to show the entire decimal because the decimal for $\sqrt{2}$ is infinite and does not repeat.

So, try to find the irrational numbers among the given numbers:

$\sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{8}$ and $\sqrt{9}$

Terminology

Decimal number – ондық бөлшек/ десятичная дробь

Integer – бүтін сан/ целое число

Irrational number – иррационал сан/ иррациональное число

Natural number – натурал сан/ натуральное число

Rational number – рационал сан/ рациональное число

Real number – нақты сан/ действительное число

Whole number – теріс емес бүтін сан/ неотрицательное целое число

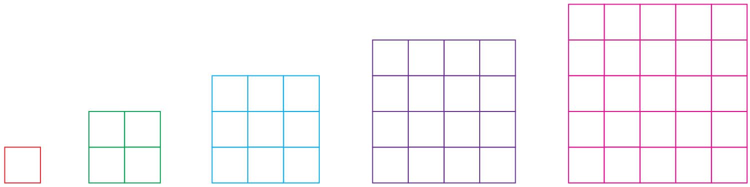
Note

Note that!

It is also true that $(-4)\cdot(-4)=16$, but a negative number is impossible in this problem.

UNDERSTANDING SQUARE ROOTS

When we express a number as the product of two equal factors, that factor is called the square root of the number.
For example,



A=1 A=4 A=9 A=16 A=25

How do you find the length of one side of a square with the given area?
First, let's show a solution for the square with an area of 16 cm^2. We want to find the length of its sides. Let's take the unknown sides of the square as *a*.
To find the side length, determine what positive number multiplied by itself, equals 16.

$$4\cdot 4=16$$

The square root of 16 is 4, or this is written $\sqrt{16}=4$.
So, we can see that subtraction is the opposite of addition, and **the square root** is the *opposite of squaring*.

Definition

If $a^2=b$ then a is the **square root** of $b\,(b\geq 0, a\geq 0)$.
We use the symbol $\sqrt{\ }$, called **radical**, to denote the square root of a number. $\sqrt{b}$ is read as '**the square root of** *b*'. So if $a^2=b$ then $a=\sqrt{b}\ (b\geq 0, a\geq 0)$.

Here are the square roots of all the perfect squares from 1 to 100.

$1^2=1 \Rightarrow \sqrt{1}=1$ $\qquad$ $6^2=36 \Rightarrow \sqrt{36}=6$

$2^2=4 \Rightarrow \sqrt{4}=2$ $\qquad$ $7^2=49 \Rightarrow \sqrt{49}=7$

$3^2=9 \Rightarrow \sqrt{9}=3$ $\qquad$ $8^2=64 \Rightarrow \sqrt{64}=8$

$4^2=16 \Rightarrow \sqrt{16}=4$ $\qquad$ $9^2=81 \Rightarrow \sqrt{81}=9$

$5^2=25 \Rightarrow \sqrt{25}=5$ $\qquad$ $10^2=100 \Rightarrow \sqrt{100}=10$

The equation $x^2=9$ can be stated as the question, 'What number multiplied itself is 9?' There are two numbers 3 and –3.

Fact

Put any positive number into your calculator. Take the square root. Continue to take the square root; eventually, you will always get 1.

Rule

If $x \in \mathbb{R}$ then

$$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

In other words, if x is a **non-negative real number**, then $\sqrt{x^2} = x$, and if x is a **negative real number**, then $\sqrt{x^2} = -x$.

For example,

$$\sqrt{3^2} = 3, \left(\sqrt{3^2} = \sqrt{9} = 3\right) \text{ and } \sqrt{(-3)^2} = -(-3), \left(\sqrt{(-3)^2} = \sqrt{9} = 3\right)$$

We can conclude that the square root of any real number will always be greater than or equal to zero. $\sqrt{-9}$ is undefined. Negative numbers have no square root because the square of any real number cannot be negative.

$$\sqrt{-9} \neq 3, \text{ since } 3^2 \text{ is } 9, \text{ not } (-9)$$
$$\sqrt{-9} \neq -3, \text{ since } (-3)^2 \text{ is } 9, \text{ not } (-9)$$

Discussion

Discuss with your friends. What is meant by a **perfect square**? Give an example.

Example 1

Evaluate each square root.

a. $\sqrt{25}$ b. $\sqrt{36}$ c. $\sqrt{81}$ d. $\sqrt{121}$

e. $\sqrt{225}$ f. $\sqrt{1}$ g. $\sqrt{400}$ h. $\sqrt{10000}$

Solution:

To simplify a square root, we remove anything that is a "perfect square". That is, you take out front anything that has two copies of the same factor:

a. $\sqrt{25} = \sqrt{5^2} = 5$ b. $\sqrt{36} = \sqrt{6^2} = 6$

c. $\sqrt{81} = \sqrt{9^2} = 9$ d. $\sqrt{121} = \sqrt{11^2} = 11$

e. $\sqrt{225} = \sqrt{15^2} = 15$ f. $\sqrt{1} = \sqrt{1^2} = 1$

g. $\sqrt{400} = \sqrt{20^2} = 20$ h. $\sqrt{10000} = \sqrt{100^2} = 100$

Activity

A student said that since the square roots of a certain number are 1.4 and –1.4, the number must be their product, –1.96. What error did the students make?

Practice 1

Evaluate each square root.

a. $\sqrt{64}$ b. $\sqrt{100}$ c. $\sqrt{144}$ d. $\sqrt{196}$

e. $\sqrt{289}$ f. $\sqrt{529}$ g. $\sqrt{841}$ h. $\sqrt{1600}$

Terminology

Square root – квадрат түбір / квадратный корень

Activity

Color each box that has a perfect square yellow. Then, find the square root. If the box does not contain a perfect square, color the box orange.

$\sqrt{100}$	$\sqrt{25}$	$\sqrt{12}$
$\sqrt{22}$	$\sqrt{10}$	$\sqrt{97}$
$\sqrt{13}$	$\sqrt{81}$	$\sqrt{36}$
$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$
$\sqrt{15}$	$\sqrt{121}$	$\sqrt{24}$
$\sqrt{45}$	$\sqrt{225}$	$\sqrt{64}$
$\sqrt{49}$	$\sqrt{10000}$	$\sqrt{32}$
$\sqrt{16}$	$\sqrt{5}$	$\sqrt{50}$

Example 2

Evaluate each square root.

a. $\sqrt{0}$ b. $\sqrt{0.16}$ c. $\sqrt{0.64}$ d. $-\sqrt{169}$

e. $-\sqrt{256}$ f. $\sqrt{-324}$ g. $\sqrt{(-19)^2}$ h. $\sqrt{-22^2}$

Solution:

a. $\sqrt{0}=0$, because $0^2=0$

b. $\sqrt{0.16}=0.4$, because $0.4^2=0.16$

c. $\sqrt{0.64}=0.8$, because $0.8^2=0.64$

d. Firstly, we find $\sqrt{169}$, and then we will multiply it by –1. So,

$$-\sqrt{169}=-1\cdot\sqrt{169}=-1\cdot\sqrt{13^2}=-1\cdot 13=-13$$

e. $-\sqrt{256}=-1\cdot\sqrt{256}=-1\cdot\sqrt{16^2}=-1\cdot 16=-16$

f. $\sqrt{-324}$ is undefined. Remember that the square of any real number cannot be negative.

g. By using the rule,

$$\sqrt{(-19)^2}=-(-19)=19 \text{ or } \sqrt{(-19)^2}=\sqrt{(-19)\cdot(-19)}=\sqrt{361}=19$$

h. $\sqrt{-22^2}=\sqrt{-(22)\cdot(22)}=\sqrt{-484}$ is undefined

Note

Remember that!

$$-22^2\neq(-22)^2=484$$

$$-22^2=-1\cdot 22^2=-484$$

Practice 2

Evaluate each square root.

a. $\sqrt{0.9}$ b. $\sqrt{0.25}$ c. $\sqrt{4.41}$ d. $-\sqrt{1296}$

e. $-\sqrt{1024}$ f. $\sqrt{-576}$ g. $\sqrt{(-41)^2}$ h. $\sqrt{-53^2}$

PROPERTIES OF SQUARE ROOTS

Property 1

For any real number *a* and *b*, where $a\geq 0$, and $b\geq 0$,

$$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b} \text{ or } \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}$$

For example,

$$\sqrt{25\cdot 16}=\sqrt{25}\cdot\sqrt{16}=5\cdot 4=20$$

$$\sqrt{3}\cdot\sqrt{27}=\sqrt{3\cdot 27}=\sqrt{81}=9$$

$$\sqrt{36a^2}=\sqrt{36}\cdot\sqrt{a^2}=6\cdot a=6a \qquad (a\geq 0)$$

Note

If $a\geq 0$ then

$$\sqrt{a}\cdot\sqrt{a}=\sqrt{a\cdot a}=\sqrt{a^2}=a$$

Example 3

Simplify each of the following.

a. $\sqrt{2}\cdot\sqrt{8}$ b. $\sqrt{5}\cdot\sqrt{5}$ c. $\sqrt{50}\cdot\sqrt{2}$ d. $\sqrt{90}\cdot\sqrt{10}$

Solution:

By using *Property 1*, we can simplify the expressions. So,

a. $\sqrt{2}\cdot\sqrt{8}=\sqrt{2\cdot 8}=\sqrt{16}=4$

b. $\sqrt{5}\cdot\sqrt{5}=\sqrt{5\cdot 5}=\sqrt{25}=5$

c. $\sqrt{50}\cdot\sqrt{2}=\sqrt{50\cdot 2}=\sqrt{100}=10$

d. $\sqrt{90}\cdot\sqrt{10}=\sqrt{90\cdot 10}=\sqrt{900}=30$

Practice 3

Evaluate each square root.

a. $\sqrt{2}\cdot\sqrt{2}$ b. $\sqrt{8}\cdot\sqrt{32}$ c. $\sqrt{48}\cdot\sqrt{3}$ d. $\sqrt{56}\cdot\sqrt{14}$

Example 4

Simplify each of the following.

a. $\sqrt{13^2-12^2}$ b. $\sqrt{36\cdot 49}$ c. $\sqrt{6\cdot 54}$ d. $-\sqrt{11\cdot 44}$

Solution:

By using *Property 1*, we can simplify the expressions. So,

a. $\sqrt{13^2-12^2}=\left|\sqrt{a^2-b^2}=\sqrt{(a-b)\cdot(a+b)}\right|=$

$=\sqrt{(13-12)\cdot(13+12)}=\sqrt{1\cdot 25}=\sqrt{25}=5$

b. $\sqrt{36\cdot 49}=\sqrt{36}\cdot\sqrt{49}=\sqrt{6^2}\cdot\sqrt{7^2}=6\cdot 7=42$

c. $\sqrt{6\cdot 54}=\sqrt{324}=\sqrt{18^2}=18$

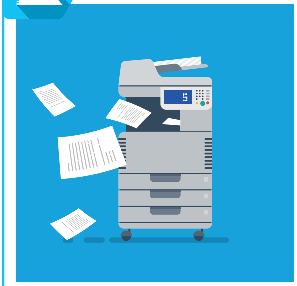
d. $-\sqrt{11\cdot 44}=-1\cdot\sqrt{484}=-1\cdot\sqrt{22^2}=-22$

Practice 4

Evaluate each square root.

a. $\sqrt{64\cdot 225}$ b. $\sqrt{0.25\cdot 196}$ c. $\sqrt{13\cdot 52}$ d. $\sqrt{17^2-8^2}$

Real life



Suppose you use a copying machine to reduce a picture for a project. Let's say you want to reduce a picture to half size, which is 50 percent. Most copiers can only reduce to 69 percent. So if you take the square root of 50 percent you get 71 percent. Set the copier to 71 percent, copy the picture once, and then copy the COPY. That's 0.71 times 0.71 equals 0.5 or 50 percent!

Terminology

Property - қасиеті/ свойства

Undefined - анықталмаған/ неопределенный

Note

If $a \geq 0$ then

$$\frac{\sqrt{a}}{\sqrt{a}} = \sqrt{\frac{a}{a}} = \sqrt{1} = 1$$

Property 2

For any real numbers a and b, where $a \geq 0$, and $b > 0$,

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ or } \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

For example,

$$\sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}} = \frac{5}{8}$$

$$\frac{\sqrt{54}}{\sqrt{6}} = \sqrt{\frac{54}{6}} = \sqrt{9} = 3$$

Example 5

Simplify each of the following.

a. $\sqrt{\frac{25}{9}}$ b. $\frac{\sqrt{50}}{\sqrt{2}}$ c. $-\sqrt{\frac{16}{121}}$ d. $\sqrt{\frac{15}{135}}$

Solution:

By using *Property 2*, we can simplify the expressions. So,

a. $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{\sqrt{5^2}}{\sqrt{3^2}} = \frac{5}{3}$

b. $\frac{\sqrt{50}}{\sqrt{2}} = \sqrt{\frac{50}{2}} = \frac{\sqrt{25}}{\sqrt{1}} = \frac{5}{1} = 5$

c. $-\sqrt{\frac{16}{121}} = -1 \cdot \frac{\sqrt{16}}{\sqrt{121}} = -1 \cdot \frac{4}{11} = -\frac{4}{11}$

d. $\sqrt{\frac{15}{135}} = \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$

Practice 5

Evaluate each square root.

a. $\sqrt{\frac{81}{9}}$ b. $\frac{\sqrt{12}}{\sqrt{3}}$ c. $-\sqrt{\frac{100}{169}}$ d. $\sqrt{\frac{2}{32}}$

Example 6

Simplify each of the following.

a. $\sqrt{\frac{5}{7}}\cdot\sqrt{\frac{63}{125}}$ b. $\sqrt{5\frac{1}{16}}$ c. $\sqrt{1\frac{11}{25}\cdot\frac{100}{49}}$

Solution:

a. At first, we use *Property 1* to multiply the square roots. So,

$$\sqrt{\frac{5}{7}}\cdot\sqrt{\frac{63}{125}}=\sqrt{\frac{5}{7}\cdot\frac{63}{125}}=\sqrt{\frac{9}{25}}$$

Then, we use *Property 2* to simplify the expressions:

$$\sqrt{\frac{9}{25}}=\frac{\sqrt{9}}{\sqrt{25}}=\frac{\sqrt{3^2}}{\sqrt{5^2}}=\frac{3}{5}$$

b. Firstly, change to improper fraction:

$$5\frac{1}{16}=\frac{5\cdot 16+1}{16}=\frac{81}{16}$$

Therefore,

$$\sqrt{5\frac{1}{16}}=\sqrt{\frac{81}{16}}=\frac{\sqrt{81}}{\sqrt{16}}=\frac{9}{4}$$

c. As we know that $1\frac{11}{25}=\frac{36}{25}$. Therefore,

$$\sqrt{1\frac{11}{25}\cdot\frac{100}{49}}=\sqrt{\frac{36}{25}\cdot\frac{100}{49}}=\frac{\sqrt{36}}{\sqrt{25}}\cdot\frac{\sqrt{100}}{\sqrt{49}}=\frac{6}{5}\cdot\frac{10}{7}=\frac{12}{7}$$

Discussion

By discussing with your friends, try to find the two-digit number with the square sum of its digits equal to the number obtained by reversing its digits.

Practice 6

Evaluate each square root.

a. $\sqrt{\frac{6}{7}}\cdot\sqrt{\frac{343}{96}}$ b. $\sqrt{2\frac{7}{9}}$ c. $\sqrt{3\frac{13}{36}\cdot\frac{49}{4}}$

Property 3

For any non-negative real number a $(a\geq 0)$ and $n\in\mathbb{N}$,

$$\left(\sqrt{a}\right)^n=\sqrt{a^n}$$

Proof:

$$\left(\sqrt{a}\right)^n=\underbrace{\sqrt{a}\cdot\sqrt{a}\cdot\sqrt{a}\cdot\ldots\cdot\sqrt{a}}_{n\text{ factors of }\sqrt{a}}=\sqrt{\underbrace{a\cdot a\cdot a\cdot\ldots\cdot a}_{n\text{ factors of }a}}=\sqrt{a^n}$$

Real life

GPS and mapping use square roots all the time to figure out where you are!

For example,

$$\left(\sqrt{2}\right)^8 = \sqrt{2^8} = \sqrt{256} = 16$$

$$\left(\sqrt{3}\right)^4 = \sqrt{3^4} = \sqrt{81} = 9$$

Example 7

Evaluate:

$$\left(\sqrt{2}\right)^4 + \left(\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^6$$

Solution:

By using *Property 3*, we can calculate the expressions. So,

$$\left(\sqrt{2}\right)^4 + \left(\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^6 = \sqrt{2^4} + \sqrt{5^4} - \sqrt{5^2} - \sqrt{2^6} =$$

$$= \sqrt{16} + \sqrt{625} - \sqrt{25} - \sqrt{64} = 4 + 25 - 5 - 8 = 16$$

Practice 7

Evaluate:

1. $\left(\sqrt{3}\right)^4 + \left(\sqrt{7}\right)^4 - \left(\sqrt{29}\right)^2 + \left(\sqrt{2}\right)^8$
2. $\left(\sqrt{11}\right)^2 + \left(\sqrt{13}\right)^4 - \left(\sqrt{9}\right)^3 + 2$
3. $\left(\sqrt{(-1)}\right)^4 - \sqrt{(-1)^2} + \left(\sqrt{2}\right)^6 - \sqrt{121}$

ESTIMATING THE VALUE OF SQUARE ROOTS

It is easy to work out the square root of a perfect square, but it is hard to work out other square roots. One way to estimate the square root of any number is to find a whole number greater than the square root and another whole number less than the square root.

Example 8

Estimate the square root of 2.

Solution:

Let's first find two consecutive natural numbers, which are smaller and bigger than $\sqrt{2}$. $1 < \sqrt{2} < 2$ because $1^2 < 2 < 2^2$, so $\sqrt{2}$ lies between 1 and 2.

For more accuracy we will try 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9.

$1.4<\sqrt{2}<1.5$ since $1.4^2=1.96<2<1.5^2=2.25$. So $\sqrt{2}$ is approximately between 1.4 and 1.5 with accuracy 1 decimal place (0.1).
In the same way

$$1.4<\sqrt{2}<1.5$$
$$1.41<\sqrt{2}<1.42$$
$$1.414<\sqrt{2}<1.415$$
$$1.4142<\sqrt{2}<1.4143$$
$$\dots$$

Practice 8

1. Estimate the square root of 17 to 3 decimal place.
2. Estimate the radical number $\sqrt{97}$ to 2 decimal place.
3. Estimate the square root of 102 to 2 decimal place.

For approximation of square root we also can use formula $\sqrt{c}=\sqrt{a^2+b}\approx a+\frac{b}{2a}$, where $\sqrt{c}$ is a given square root and a^2 is the nearest perfect square to c.

Example 9

Calculate the approximation of square root 28.

Solution:

By the formula

$$\sqrt{28}=\sqrt{5^2+3}\approx 5+\frac{3}{10}=5.3$$

Practice 9

1. Calculate the approximation of square root 50.
2. Calculate the approximation of square root 1700.

Terminology

Estimating the value of square roots - квадраттық түбірлер мәнің бағалау/ оценка значений квадратных корней

PROBLEMS 1.1

A

1. Which of these sets does not contain rational numbers.

a. $13, \frac{1}{3}, -14$

b. $-4,6042..., \sqrt{98}, 0.243874...$

c. $-7, \sqrt{225}, 4\frac{7}{9}$

d. $\sqrt{81}, 0.75, 0$

2. What is the best classification for –6?

a. whole number, integer, real number

b. integer, rational number, real number

c. rational number, real number

d. irrational number, real number

3. Which of the following square roots is an irrational number?

a. $-\sqrt{25}$ b. $\sqrt{49}$ c. $\sqrt{12}$ d. $\sqrt{\frac{1}{16}}$

4. Evaluate the square roots.

a. $\sqrt{49}$ b. $\sqrt{144}$ c. $\sqrt{256}$

d. $\sqrt{441}$ e. $\sqrt{676}$ f. $\sqrt{-100}$

g. $\sqrt{2.89}$ h. $\sqrt{1.21}$ i. $\sqrt{20.25}$

k. $\sqrt{(-23)^2}$ l. $-\sqrt{-1600}$ m. $-\sqrt{1764}$

5. Simplify the expressions.

a. $\sqrt{49 \cdot 64}$ b. $\sqrt{81 \cdot 225}$

c. $\sqrt{1.96 \cdot 961}$ d. $\sqrt{2.25 \cdot 100}$

e. $\sqrt{16 \cdot 0.16 \cdot 144}$ f. $\sqrt{1.96 \cdot 0.04 \cdot 900}$

6. Simplify the expressions.

a. $\sqrt{117^2 - 108^2}$ b. $\sqrt{122^2 - 22^2}$

c. $\sqrt{8^2 + 4^2}$ d. $\sqrt{442^2 - 42^2}$

e. $\sqrt{128} \cdot \sqrt{32}$ f. $\sqrt{12} \cdot \sqrt{48}$

g. $\sqrt{2.2} \cdot \sqrt{8.8}$ h. $\sqrt{98} \cdot \sqrt{4.5}$

7. Evaluate each square root.

a. $\sqrt{\frac{49}{81}}$ b. $-\sqrt{\frac{225}{841}}$

c. $\sqrt{5\frac{1}{16}}$ d. $\sqrt{\frac{256}{25}}$

e. $-\sqrt{\frac{36}{81} \cdot \frac{144}{4}}$ f. $\sqrt{2.25 \cdot \frac{16}{49} \cdot 1.69}$

8. Evaluate each square root.

a. $\frac{\sqrt{2}}{\sqrt{162}}$ b. $\frac{\sqrt{242}}{\sqrt{32}}$

c. $\sqrt{\left(\frac{13}{36}\right)^2 - \left(\frac{12}{36}\right)^2}$ d. $\sqrt{\left(\frac{1.22}{25}\right)^2 - \left(\frac{0.22}{25}\right)^2}$

9. Evaluate

a. $\left(\sqrt{8}\right)^2$ b. $\left(-\sqrt{5}\right)^2$

c. $-\left(\sqrt{3}\right)^2$ d. $\left(2\sqrt{5}\right)^2$

e. $\left(\sqrt{(-6)^2}\right)^2$ f. $-\left(-\sqrt{(-5)^2}\right)^2$

10. Solve the equation, if $x > 0$.

a. $x^2 = 49$ b. $x^2 = 64$

c. $x^2 = 0.04$ d. $x^2 = 625$

e. $x^2 = 121$ f. $x^2 = 36$

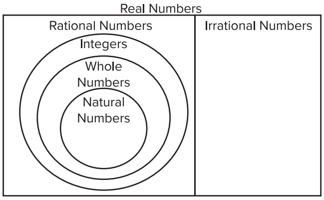
11. Estimate each value to the nearest hundreds.

a. $\sqrt{2}$ b. $\sqrt{8}$ c. $\sqrt{11}$ d. $\sqrt{3}$ e. $\sqrt{5}$

B

12. Classifying Real numbers. Write each number in the correct location on the Venn Diagram of the real number system. Each number should be written only once.

$$\frac{-10}{2}, \sqrt{36}, \frac{0}{8}, 7, \sqrt{140}, \frac{4}{9}, \sqrt{4}, -8, \sqrt{8}, -2.89$$



13. Calculate.

a. $4\cdot\sqrt{11}\cdot\sqrt{99}$ b. $4.25-\sqrt{0.0625}$

c. $\sqrt{0.16}+5.6$ d. $-0.4\cdot\sqrt{225}+2\frac{1}{2}\cdot\sqrt{5.76}$

14. Simplify the expressions.

a. $\frac{\sqrt{6}}{\sqrt{150}}+\sqrt{0.64}$ b. $\frac{\sqrt{54}}{\sqrt{96}}-\sqrt{144}$

c. $10\cdot\sqrt{\frac{361}{100}}-\sqrt{81}$ d. $5\sqrt{64}+\left(-\sqrt{8}\right)^2$

e. $4\sqrt{81}-\left(-\sqrt{5}\right)^2$ f. $\left(\sqrt{2}\right)^4+\sqrt{72}\cdot\sqrt{8}$

15. Estimate each number nearest to ones.

a. $\sqrt{123}$ b. $\sqrt{38}$ c. $\sqrt{624}$

d. $\sqrt{399}$ e. $\sqrt{742}$

C

16. Given that $\sqrt{2+2}=2$, does $\sqrt{a+a}=a$? Explain.

17. Calculate.

a. $\sqrt{14.4+\sqrt{121}-\sqrt{0.16}}$

b. $\sqrt{\sqrt{784}-\sqrt{2.56}-\sqrt{1.69}-\sqrt{0.01}}$

c. $\sqrt{-z^2+k^3}$, if $z=-30$ and $k=10$

d. Determine the value of y, if $\sqrt{\frac{y}{5}}=\frac{2}{5}$

e. $\sqrt{\frac{z^2-k^2}{z+k}}$, if $z=19$ and $k=10$

18. Simplify the expressions.

a. $\sqrt{\frac{66\cdot\sqrt{50^2-40^2}}{\sqrt{44^2+33^2}}}$

b. $\sqrt{(-6)^4:(0.6)^6}\cdot\left(\frac{6}{10^2}\right)$

19. Solve the equations (x>0, y>0).

a. $(x-3.5)^2=49$ b. $(x-6)^2=64$

c. $(3-2x)^2=0.04$ d. $(5y-1)^2=625$

e. $(x-3)^2=121$ f. $(x-4)^2=36$

20. Simplify the expressions.

a. $\frac{\sqrt{-64+\left(4\sqrt{5}\right)^2}}{\sqrt{145^2-130^2}}\cdot\frac{15\sqrt{15}\cdot\sqrt{275}}{4}\cdot\frac{\sqrt{16}}{\sqrt{225}}$

b. $\sqrt{(6-10)^2}\cdot\frac{\sqrt{128^2-41^2}}{\sqrt{156^2-69^2}}\cdot\frac{15}{13}$

c. $\sqrt{725-14^2}\cdot\frac{\sqrt{15004+5^3}}{123}$

d. $\sqrt{15^2-81}\cdot\sqrt{\frac{0.73^2-0.57^2}{0.67^2-0.63^2}}\cdot\frac{1}{\sqrt{4}}$

21. Estimate each number nearest to ones.

a. $\sqrt{1023}$ b. $\sqrt{2003}$ c. $\sqrt{2019}$

d. $\sqrt{1993}$ e. $\sqrt{5861}$