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PREFACE

Algebra 9 is designed to provide learners to improve and develop their mathematical background needed for further algebra courses.

Starting from the first pages, you will realize that this textbook is completely different from any other usual textbook full of theoretical passages and formulas. Every chapter contains useful information, curious facts, tasks for individual and group work. You will also learn how to conduct research and experiments yourselves, search for information, make your discoveries. By the end of the book, students will have a good understanding of the analytic approach to solving problems. In addition, we have provided many systematic explanations throughout the text that will help instructors to reach the goals that they have set for their students.

One more valuable feature of this textbook is the language. Every sentence has been carefully chosen so that it is not difficult for you to understand mathematics in the English language. Each page contains translations of all the important terms, both in Kazakh and Russian. This textbook will not only help you improve your English, but it will also make you a part of a big international science community.

This textbook consists of five chapters, which cover EQUATIONS. INEQUALITIES IN TWO VARIABLES AND THIER SYSTEMS, ELEMENTS OF COMBINATORICS, NUMBER SEQUENCES, TRIGONOMETRY, ELEMENTS OF PROBABILITY THEORY respectively. Each chapter begins with basic definitions, theorems, and explanations which are necessary for understanding the subsequent chapter material. In addition, each chapter is divided into subsections so that students can follow the material easily.

Every subsection includes self-test PRACTICE problem sections. Teachers should encourage their students to solve PRACTICE problems themselves because these problems are fundamental to understanding and learning the related subjects or sections. At the end of every section, there are PROBLEMS categorized according to the structure and subject matter of the section. Problems are graded in order, from easy (A) to difficult (C). In addition, at the end of every chapter there is SUMMARY categorized according to the structure and subject matter of the chapter.

Please pay attention to the structure of this textbook. Remember: a textbook is no longer the only source of information in the modern world. With the help of carefully selected tasks, you are going to learn such important skills as critical thinking, problem solving, information analysis, creativity, imagination, teamwork, digital literacy, etc.

If you have any questions, suggestions or ideas regarding the contents of this book, please feel free to contact us:

- via email: admin@astanakitap.kz

Best regards,,

team of authors, "Астана-кітап"

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Chapter 1

EQUATIONS. INEQUALITIES IN TWO VARIABLES AND THEIR SYSTEMS

1.1 NON-LINEAR EQUATIONS IN TWO VARIABLES AND THEIR SYSTEMS

You will:

- distinguish linear and non-linear equations in two variables;
- solve systems of non-linear equations in two variables;
- solve written problems using systems of equations;
- make mathematical model according to given in an exercise;

LINEAR EQUATIONS IN TWO VARIABLES

You are already familiar with terms like linear equations in one variable, inequality in one variable. In this chapter we will study equations in two variables and inequalities in two variables.

The equation $x+y=5$ has two variables x and y. The solution to the equation is an ordered pair of numbers, (x,y). This type of equation is called a linear equation in two variables. Let us recall the definition of equation in two variables.

Definition

A first degree equation is called a **linear equation in two variables** if it contains two distinct variables.

Look at some examples of linear equations in two variables:

$3x+2y=10$, $5m+4=6-2n$ and $2a-b=7$.

Example 1

Given $y=x+4$, find

a. y when $x=2$

b. x when $y=8$

Solution:

a. If $x=2$, then

$$y=(2)+4$$
$$y=6$$

b. If $y=8$, then

$$(8)=x+4$$
$$x=4$$

Practice 1

Given $y=x-5$, find

a. y when $x=7$

b. x when $y=10$

Example 2

Given $2y=3x-7$, find

a. y when $x=3$

b. x when $y=4$

Solution:

a. If $x=3$, then

$$2y = 3\cdot(3) - 7$$
$$2y = 9 - 7$$
$$2y = 2$$
$$y = 1$$

b. If $y = 8$, then

$$2\cdot(4) = 3x - 7$$
$$8 = 3x - 7$$
$$3x = 8 + 7$$
$$3x = 15$$
$$x = 3$$

Practice 2

Given $3y = 2x - 5$, find

a. y when $x = 4$

b. x when $y = 5$

Example 3

Given $\frac{5}{18}y = -\frac{2}{5}x + \frac{4}{9}$, find

a. y when $x = 15$

b. x when $y = 36$

Solution:

a. If $x = 15$, then

$$\frac{5}{18}y = -\frac{2}{5}\cdot 15 + \frac{4}{9}$$
$$\frac{5}{18}y = -6 + \frac{4}{9}$$
$$\frac{5}{18}y = -\frac{50}{9}$$
$$y = -20$$

b. If $y = 36$, then

$$\frac{5}{18}\cdot 36 = -\frac{2}{5}x + \frac{4}{9}$$
$$10 = -\frac{2}{5}x + \frac{4}{9}$$
$$\frac{2}{5}x = -\frac{86}{9}$$
$$x = -\frac{215}{9}$$

Terminology

Distinguish – ажырату/различать

Ordered pair – реттелген жұп/упорядоченная пара

Variable – өзгермелі/переменная

Practice 3

Given $-\frac{2}{9}y = \frac{2}{3}x + \frac{4}{9}$, find

a. y when $x = 9$

b. x when $y = 18$

Example 4

Are the ordered pairs $(2, 1)$ and $(-4, 6)$ solutions of $y = 3x - 5$?

Solution:

$$(2, 1) \Rightarrow x = 2 \text{ and } y = 1$$
$$1 = 3 \cdot (2) - 5$$
$$1 = 6 - 5$$
$$1 = 1$$

This is true, so $(2, 1)$ is a solution.

$$(-4, 6) \Rightarrow x = -4 \text{ and } y = 6$$
$$6 = 3 \cdot (-4) - 5$$
$$6 = -12 - 5$$
$$6 \neq -17$$

This is false, so $(-4, 6)$ is not a solution.

Practice 4

Are the ordered pairs $(1, 9)$ and $(-1, 3)$ solutions of $y = 4x + 7$?

Example 6

Are the ordered pairs $(11, 6)$ and $(-22, 1)$ solutions of $-\frac{2}{11}x = \frac{19}{6}y + \frac{5}{6}$?

Solution:

$$(11, 6) \Rightarrow x = 1 \text{ and } y = 3$$
$$-\frac{2}{11} \cdot 11 = \frac{19}{6} \cdot 6 + \frac{5}{6}$$
$$-2 = 1 + \frac{5}{6}$$
$$-2 \neq \frac{11}{6}$$

This is false, so $(11, 6)$ is not a solution.

$$\left(-22, 1\right) \Rightarrow x = -22 \text{ and } y = 1$$

$$-\frac{2}{11}\cdot\left(-22\right) = \frac{19}{6}\cdot 1 + \frac{5}{6}$$

$$-2\cdot\left(-2\right) = \frac{19+5}{6}$$

$$4 = 4$$

This is true, so $\left(-22, 1\right)$ is a solution.

Practice 6

Are the ordered pairs $\left(-24, 7\right)$ and $\left(16, -6\right)$ solutions of $\frac{13}{8}y = -\frac{5}{8}x + \frac{1}{4}$?

NONLINEAR EQUATIONS IN TWO VARIABLES

Definition

Equation whose graph does not form a straight line (linear) is called a **nonlinear equation in two variables** if it contains two distinct variables.

Look at some examples of nonlinear equations in two variables:

$5x^2 + 3y = 12$, $m^3 - 7mn + n^5 = 9$, $a^6 - 8b^3 + a^3b^4 = 25$ and so on.

The degree of a nonlinear equation in two variables is the same as the highest degree of the terms in the equation.

For example,

$5x^2 + 3y = 12$ is a second degree equation.

$m^3 - 7mn + n^5 = 9$ is a fifth degree equation.

$a^6 - 8b^3 + a^3b^4 = 25$ is a seventh degree equation.

Example 7

Find the degree of the following nonlinear equations.

a. $4x^2 - 7xy = 9$

b. $x^2 - y^2 = 25$

c. $x^2y^2 - y = 12$

d. $x^7 + 5y^3 = 6xy$

e. $x^4y^5 + 8x^3y^2 = 18$

Solution:

Recall that the degree of the monomial is the sum of the exponents of its variables. The degree of a polynomial is the highest degree of its terms. According to this

Terminology

Nonlinear equation – сызықты емес теңдеу/ нелинейное уравнение

Straight – түзу/ прямая

Equation	Degree
$4x^2-7xy=9$	2
$x^2-y^2=25$	2
$x^2y^2-y=12$	4
$x^7+5y^3=6xy$	7
$x^4y^5+8x^3y^2=18$	9

Practice 7

Find the degree of the following nonlinear equations.

a. $7x^3+3xy=12$

b. $x^2-y^2=49$

c. $x^3y^6+x=8$

d. $x^8-4y^4=7xy$

e. $xy^5+8x^4y^3=18$

Example 8

Write the equations of circles with centers $O(a, b)$ and and radius R given below.

a. $O(2, 4)$, $R=4$

b. $O(5, -3)$, $R=6$

c. $O(-4, -7)$, $R=2{,}5$

d. $O(0, 6)$, $R=\sqrt{11}$

Note

The equation of a circle with center $O(a, b)$ and radius R is $(x-a)^2+(y-b)^2=r^2$.

Solution:

a. $(x-2)^2+(y-4)^2=4^2$ or $(x-2)^2+(y-4)^2=16$

b. $(x-5)^2+(y-(-3))^2=6^2 \Rightarrow (x-5)^2+(y-(-3))^2=36$

c. $(x-(-4))^2+(y-(-7))^2=2{,}5^2 \Rightarrow (x+4)^2+(y+7)^2=6{,}25$

d. $(x-0)^2+(y-6)^2=\sqrt{11}^2 \Rightarrow x^2+(y-6)^2=11$

Practice 8

Write the equations of circles with centers $O(a, b)$ and and radius R given below.

a. $O(3, 5)$, $R=3$

b. $O(-1, 7)$, $R=5$

c. $O(-5, -6)$, $R=1{,}5$

d. $O(8, 0)$, $R=\sqrt{7}$

Example 9

For which values of n point $S(2, 4)$ is on parabola $nx^2 = 8 + 3y$.

Solution:

Since point $S(2, 4)$ satisfies the equation $nx^2 = 8 + 3y$, we simply substitute 2 and 4 instead of x and y respectively.

$$n \cdot 2^2 = 8 + 3 \cdot 4$$
$$4n = 20$$
$$n = 5$$

Practice 9

For which values of m point $B(2, 1)$ is on parabola $mx^2 = 10 - 2y$.

Example 10

For which values of z point $D(0.8, 3)$ is on parabola $zx^2 = 0,6 + 1,3y$.

Solution:

Since point $D(0.8, 3)$ satisfies the equation $zx^2 = -0,06 + 1,3y$, we simply substitute 0.8 and 3 instead of x and y respectively.

$$z \cdot 0,8^2 = -0,06 + 1,3 \cdot 3$$
$$0,64z = 3,84$$
$$z = 6$$

Practice 10

For which values of t point $B(1.5, -3)$ is on parabola $tx^2 = 1,2y + 8,1$.

SYSTEMS OF NONLINEAR EQUATIONS IN TWO VARIABLES

A system of two nonlinear equations in two variables, also called a nonlinear system, contains at least one equation that can be expressed in the form $Ax + By = C$.

For examples:

$\begin{cases} x^3 + y^3 = 64 \\ x + y = 4 \end{cases}$, $\begin{cases} x^2 + y = 5 \\ x^2 - y^2 = 3 \end{cases}$ and so on.

A **solution** of a nonlinear system in two variables is an ordered pair of real numbers that satisfies both equations in the system. The **solution set** of the system is the set of all such ordered pairs. As with linear systems in two variables, the solution of a nonlinear system (if there is one) corresponds to the intersection point(s) of the graphs of the equations in the system. Unlike linear systems, the graphs can be circles, parabolas, or anything other than two lines. We will solve nonlinear systems using the Substitution method and the Elimination method.

A. The Substitution Method

The substitution method involves converting a nonlinear system into one equation in one variable by an appropriate substitution. In order to solve a system of equations using the substitution method, follow the steps:

1. Solve one of the equations for one variable in terms of the other variable.
2. Substitute the resulting expression into the other equation and solve.
3. Find the other variable by substituting the result of step 2 into the original equation.
4. Check the proposed solutions in both of the system's given equations.

Example 11

Solve the system of equations

$$\begin{cases} x+y=6 \\ x^2+3y=22 \end{cases}$$

Solution:

Step 1: Solve one of the equations for one variable in terms of the other variable. We begin by isolating one of the variables raised to the first power in either of the equations. We will solve the first equation for y:

$$x+y=6$$
$$y=6-x$$

Step 2: Substitute the resulting expression into the other equation and solve. We substitute $6-x$ for y in the second equation and solve for x:

$$x^2+3\cdot(6-x)=22$$
$$x^2+18-3x=22$$
$$x^2-3x-4=0$$
$$x_1=-1 \text{ or } x_2=4$$

Step 3: Find the other variable by substituting the result of step 2 into the original equation. Now we can find y by substituting $x_1 = -1$ and $x_2 = 4$ into $y = 6 - x$.

If $x_1 = -1$, so y_1 is

$$y = 6 - x$$
$$y = 6 - (-1)$$
$$y = 6 + 1$$
$$y_1 = 7$$

If $x_2 = 4$, so y_2 is

$$y = 6 - x$$
$$y = 6 - 4$$
$$y_2 = 2$$

So the solutions are $(-1, 7)$ and $(4, 2)$.

Step 4: Check the proposed solutions in both of the system's given equations. We begin by checking $(-1, 7)$. Replace x with –1 and y with 7.

First equation

$$x + y = 6$$
$$-1 + 7 = 6$$
$$6 = 6$$

Second equation

$$x^2 + 3y = 22$$
$$(-1)^2 + 3 \cdot 7 = 22$$
$$1 + 21 = 22$$
$$22 = 22$$

The ordered pair $(-1, 7)$ satisfies both equations. Thus, is a solution of the system. Now let's check $(4, 2)$. Replace x with 4 and y with 2.

First equation

$$x + y = 6$$
$$4 + 2 = 6$$
$$6 = 6$$

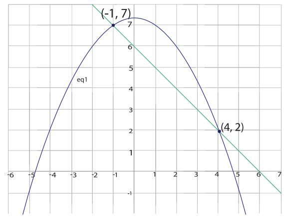
Second equation

$$x^2 + 3y = 22$$
$$4^2 + 3 \cdot 2 = 22$$
$$16 + 6 = 22$$
$$22 = 22$$

The ordered pair $(-1, 7)$ also satisfies both equations and is a solution of the system. The solutions are $(-1, 7)$ and $(4, 2)$.

Note

Graphically, we can think of the solution to the system as the points of intersections between the linear function $x + y = 6$ and quadratic function $x^2 + 3y = 22$.



Practice 11

Solve the system of equations

$$\begin{cases} 3x + y = 7 \\ x^2 + 2y = 6 \end{cases}$$

Terminology

Elimination method – қосу әдісі/ метод сложения

Substitution method – алмастыру әдісі/ метод замены

Example 12

Solve the system of equations

$$\begin{cases} 2x + y = 2 \\ 2x^2 + xy = 10 \end{cases}$$

Solution:

Step 1: Solve one of the equations for one variable in terms of the other variable. Solve the first equation for y:

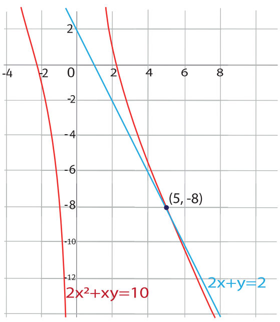
$$2x + y = 2$$
$$y = 2 - 2x$$

Step 2: Substitute the resulting expression into the other equation and solve. Substitute $2-2x$ for y in the second equation and solve for x:

$$2x^2 + x\cdot(2-2x) = 10$$
$$2x^2 + 2x - 2x^2 = 10$$
$$2x = 10$$
$$x = 5$$

Note

In the figure below, shows the graphs of the equations in the system and the solution as intersection point.



Step 3: Find the other variable by substituting the result of step 2 into the original equation. Now we can find y by substituting $x = 5$ into $y = 2 - 2x$.

$$y = 2 - 2x$$
$$y = 2 - 2\cdot 5$$
$$y = 2 - 10$$
$$y = -8$$

So the solution is $(5, -8)$.

Step 4: Check the proposed solution in both of the system's given equations. Let's check $(5, -8)$. Replace x with 5 and y with –8.

First equation	Second equation
$2\cdot x + y = 2$	$2\cdot x^2 + x\cdot y = 10$
$2\cdot 5 + (-8) = 2$	$2\cdot 5^2 + 5\cdot 8 = 10$
$10 - 8 = 2$	$50 - 40 = 10$
$2 = 2$	$10 = 10$

The ordered pair $(5, -8)$ satisfies both equations. Thus, is a solution of the system.

Practice 12

Solve the system of equations.

a. $\begin{cases} x-2y=6 \\ x^2-xy=8 \end{cases}$ b. $\begin{cases} x-y=3 \\ (x-2)^2+(y+3)^2=4 \end{cases}$

B. The Elimination Method

To solve the system of nonlinear equations we the same steps when we solve the system of linear equations in two variables.

Example 13

Solve the system of equations

$$\begin{cases} x^2-5y=9 \\ x+y=3 \end{cases}$$

Solution:

Let's eliminate y. First, we multiply the second equation by and add obtain equation to the first equation side by side:

$$\begin{cases} x^2-5y=9 \\ \color{red}{5}\cdot(x+y)=\color{red}{5}\cdot 3 \end{cases}$$

$$+\begin{cases} x^2-5y=9 \\ 5x+5y=15 \end{cases}$$

$$x^2+5x=24$$

$$x^2+5x-24=0$$

$$(x+8)\cdot(x-3)=0 \qquad \text{By Factoring Method}$$

$$x^2+5x=24$$

$$x_1=-8 \text{ or } x_2=3$$

Substitute $x_1=-8$ and $x_2=3$ into the second equation:

- If $x_1=-8$, y_1 is

$$x_1=-8 \Rightarrow -8+y=3 \Rightarrow y_1=11$$

- If $x_2=3$, y_2 is

$$x_2=3 \Rightarrow 3+y=3 \Rightarrow y_2=0$$

So the solution set is two points $(-8, 11)$ and $(3, 0)$.

Practice 13

Solve the system of equations.

$$\begin{cases} 3x^2+y=0 \\ 3x-y=6 \end{cases}$$

Example 14

Solve the system of equations

$$\begin{cases} x^2 - y^2 = 20 \\ x + y^2 = 10 \end{cases}$$

Solution:

Let's eliminate y^2. First, we multiply the second equation by and add obtain equation to the first equation side by side:

$$+\begin{cases} x^2 - y^2 = 20 \\ x + y^2 = 10 \end{cases}$$

$$\begin{aligned} &x^2 + x = 30 \\ &x^2 + x - 30 = 0 \\ &(x+6)\cdot(x-5) = 0 \qquad \text{By Factoring Method} \\ &x_1 = -6 \ \text{ or } \ x_2 = 5 \end{aligned}$$

Substitute $x_1 = -6$ and $x_2 = 5$ into the second equation:

- If $x_1 = -6$, y_1 is

$$x_1 = -6 \Rightarrow -6 + y^2 = 10 \Rightarrow y_1 = -4; y_2 = 4$$

- If $x_2 = 5$, y_2 is

$$x_2 = 5 \Rightarrow 5 + y^2 = 10 \Rightarrow y_3 = -\sqrt{5}; y_4 = \sqrt{5}$$

So the solution set is four points $(-6, -4)$, $(-6, 4)$, $(5, -\sqrt{5})$ and $(5, \sqrt{5})$.

Practice 14

Solve the system of equations.

$$\begin{cases} x^2 + y^2 = 40 \\ x - y^2 = 2 \end{cases}$$

Example 15

Solve the system of equations

$$\begin{cases} x^2 + xy = 2 \\ y - 2x = 5 \end{cases}$$

Solution:

Let's eliminate xy. First, we multiply the second equation by $-x$ and add obtained equation to the first equation side by side:

$$\begin{cases} x^2 + xy = 2 \\ -x \cdot \left(y - 2x^2\right) = -x \cdot 5 \end{cases}$$

$$+\begin{cases} x^2 + xy = 2 \\ 2x^2 - xy = -5x \end{cases}$$

$$3x^2 = 2 - 5x$$

$3x^2 + 5x - 2 = 0$ Standard form

$(x+2)\cdot(3x-1) = 0$ By Factoring Method

$$x_1 = -2 \text{ or } x_2 = \frac{1}{3}$$

Substitute $x_1 = -2$ and $x_2 = \frac{1}{3}$ into the second equation:

- If $x_1 = -2$, y_1 is

$$x_1 = -2 \Rightarrow y - 2\cdot(-2) = 5 \Rightarrow y_1 = 1$$

- If $x_2 = \frac{1}{3}$, y_2 is

$$x_2 = \frac{1}{3} \Rightarrow y - 2\cdot\frac{1}{3} = 5 \Rightarrow y_2 = \frac{17}{3}$$

So the solution set is two points $(-2, 9)$ and $\left(\frac{1}{3}, \frac{17}{3}\right)$.

Practice 15

Solve the system of equations.

$$\begin{cases} x^2 - xy = 10 \\ y + 4x = 5 \end{cases}$$

WRITTEN PROBLEMS USING SYSTEMS OF EQUATIONS

After studying nonlinear equations you will be able to use the methods you have learned to solve written problems using systems of equations.

In order to solve a written problems using the systems of nonlinear equations, follow the steps:

1. Make mathematical model

2. Work with mathematical model and answer to question task

Activity

Askar wants his parents to buy a cell phone for 70000 tenge in after two quarters. Each quarter has 12 school subjects. Parents set a condition for him, if in the quarter "5" comes out for a quarter, then for this item they will give 5000 tenge, and if there is a rating of "5" in more than ten items, then they will provide a bonus of 10000 tenge. How many school subjects he needs to dial to buy a cell phone?

Terminology

Written Problems
- мәтін есептер/
текстовые задачи

Example 16

A number has two digits whose sum is 13 and sum of the squares this digits are 97. Find the number.

Solution:

Step 1: Make mathematical model. $\overline{xy}$ is unknown number and x, y are digits of this number. We can create a mathematical model of problem using the system of nonlinear equations.

$$\begin{cases} x+y=13 \\ x^2+y^2=97 \end{cases}$$

Step 2: Work with mathematical model and answer to question task. Solve the first equation for y:

$$x+y=13$$
$$y=13-x$$

Substitute $13-x$ for y in the second equation and solve for x:

$$x^2+(13-x)^2=97$$
$$x^2+169-26x+x^2=97$$
$$2x^2-26x+72=0$$
$$x^2-13x+36=0$$
$$(x-4)\cdot(x-9)=0 \qquad \text{By Factoring Method}$$
$$x_1=4 \text{ or } x_2=9$$

Substitute $x_1=4$ and $x_2=9$ into the first equation:

- If $x_1=4$, y_1 is

$$x_1+y=13$$
$$4+y=13$$
$$y_1=9$$

- If $x_2=9$, y_2 is

$$x_2+y=13$$
$$9+y=13$$
$$y_2=4$$

So the solution set is two points $(4, 9)$ and $(9, 4)$.

Practice 16

A number has to digits whose sum is 8 and sum of the squares this digits are 40. Find the number.

Example 17

Difference of length of legs of right triangle is 7 and hypotenuse is 13. Find the length of legs of right triangle.

Solution:

Step 1: Make mathematical model. We can create a mathematical model of problem using the system of nonlinear equations.

$$\begin{cases} a-b=7 \\ a^2+b^2=13^2 \end{cases}$$

Step 2: Work with mathematical model and answer to question task. Solve the first equation for a:

$$a-b=7$$
$$a=7+b$$

Substitute $7+b$ for a in the second equation and solve for b:

$$(7+b)^2+b^2=13^2$$
$$49+14b+b^2+b^2=169$$
$$2b^2+14b-120=0$$
$$b^2+7b-60=0$$
$$(b+12)\cdot(b-5)=0 \qquad \text{By Factoring Method}$$
$$b_1=-12 \text{ or } b_2=5$$

As we know, the length of the triangle can not be a negative number, so we substitute just $b=5$ into the first equation:

- If $b=5$, a is

$$a-b=7$$
$$a-5=7$$
$$a=12$$

So the solution set is $(12, 5)$.

Note

Pythagorean Theorem

$$a^2+b^2=c^2$$

where,

a, b - legs of right triangle

c - hypotenuse of right triangle

Practice 17

1. Difference of length of legs of right triamgle is 4 and hypotenuse is 20. Find the length of legs of right triangle.

2. The hypotenuse of a right triangle is 37 cm, area of right triangle is 210 cm^2. Find the length of the legs.

Example 18

The denominator of a fraction is 17 less than square of the numerator. If add 3 to the numerator and denominator, then the value of the fraction will be greater $\frac{1}{3}$. If to the numerator and denominator subtract 5, then the value of the fraction will be equal $\frac{1}{14}$.

Solution:

Step 1: Make mathematical model. x – numerator of a fraction; y – denominator of a fraction. According to the given:

$$\begin{cases} y = x^2 - 17 \\ \dfrac{x-5}{y-5} = \dfrac{1}{14} \end{cases}$$

Step 2: Work with mathematical model and answer to question task. Substitute $x^2 - 17$ for y in the second equation and solve for x:

$$\begin{cases} y = x^2 - 17 \\ \dfrac{x-5}{y-5} = \dfrac{1}{14} \end{cases} \Rightarrow \begin{cases} y = x^2 - 17 \\ y = 14x - 65 \end{cases}$$

$$x^2 - 14x + 45 = 0$$

$$(x-5)\cdot(x-9) = 0$$

$$x_1 = 5, x_2 = 9$$

And we substitute $x_1 = 5$ and $x_2 = 9$ into the first equation:

- If $x_1 = 5$, y_1 is

$$y = x^2 - 17$$

$$y = 5^2 - 17$$

$$y_1 = 8$$

- If $x_2 = 9$, y_2 is

$$y = x^2 - 17$$

$$y = 9^2 - 17$$

$$y_2 = 64$$

Additionally known $\frac{x+3}{y+3} > \frac{1}{3}$. Hence, solution $(9, 64)$ for does not satisfy this inequality. Desired fraction is $\frac{5}{8}$.

Practice 18

The denominator of a fraction less 1 square of the numerator. If to the numerator and denominator add 3, then the value of the fraction will be less $\frac{1}{3}$. If to the numerator and denominator subtract 2, then the value of the fraction will be equal $\frac{1}{10}$. Find this fraction.