

Recommended by The Ministry of Education of Republic of Kazakhstan

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PREFACE

Algebra 7 is designed to provide learners to improve and develop their mathematical background needed for further algebra courses.

Starting from the first pages, you will realize that this textbook is completely different from any other usual textbook full of theoretical passages and formulas. Every chapter contains useful information, curious facts, tasks for individual and group work. You will also learn how to conduct research and experiments yourselves, search for information, make your discoveries. By the end of the book, students will have a good understanding of the analytic approach to solving problems. In addition, we have provided many systematic explanations throughout the text that will help instructors to reach the goals that they have set for their students.

One more valuable feature of this textbook is the language. Every sentence has been carefully chosen so that it is not difficult for you to understand mathematics in the English language. Each page contains translations of all the important terms, both in Kazakh and Russian. This textbook will not only help you improve your English, but it will also make you a part of a big international science community.

This textbook consists of six chapters, which cover *NATURAL EXPONENTS, POLYNOMIALS, FUNCTIONS, ELEMENTS OF STATISTICS, BASIC IDENTITIES, RATIONAL EXPRESSIONS* respectively. Each chapter begins with basic definitions, theorems, and explanations which are necessary for understanding the subsequent chapter material. In addition, each chapter is divided into subsections so that students can follow the material easily.

Every subsection includes self-test PRACTICE problem sections. Teachers should encourage their students to solve PRACTICE problems themselves because these problems are fundamental to understanding and learning the related subjects or sections. At the end of every section, there are PROBLEMS categorized according to the structure and subject matter of the section. Problems are graded in order, from easy (A) to difficult (C). In addition, at the end of every chapter there are SUMMATIVE TEST, REAL-LIFE APPLICATION and SUMMARY categorized according to the structure and subject matter of the chapter.

Please pay attention to the structure of this textbook. Remember: a textbook is no longer the only source of information in the modern world. With the help of carefully selected tasks, you are going to learn such important skills as critical thinking, problem solving, information analysis, creativity, imagination, teamwork, digital literacy, etc.

If you have any questions, suggestions or ideas regarding the contents of this book, please feel free to contact us:

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*Best regards,
team of authors, "Астана-кітап"*

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1

CHAPTER

INTEGER EXPONENTS



1.1 THE NATURAL EXPONENTS
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1.10 USING SCIENTIFIC NOTATION
1.11 APPROXIMATION
1.12 APPROXIMATION ERRORS
1.13 APPLYING SCIENTIFIC NOTATION TO PROBLEM SOLVING

WHY STUDY INTEGER EXPONENTS

Just like it is convenient to use pencils when they are gathered in one place, the exponent helps us work with repeated multiplication of numbers. Because exponents are just a shorthand way of indicating repeated multiplication of the same thing by itself.

YOU ALREADY KNOW

- square and cube of numbers;
- relationships between units (length, mass, time, area);
- rules for finding the perimeter and area of a rectangle (square), the volume of a rectangular parallelepiped (cube);
- using the natural numbers;
- calculate using a calculator;
- order of operations (BODMAS) and evaluating expressions;
- Problem solving with arithmetic;
- rounding rule for numbers;
- determining percentage;
- determining of an even number;
- determining of an odd number;

EXERCISES 1.1

A

1. What is the difference between $8a$ and a^8 ?

2. Identify the base and exponent in each of the following.

a. 9^8

b. $(0.07)^7$

c. $(15b)^4$

d. $(8a)^9$

e. $(-6)^5$

f. $(-11x)^2$

g. $7y^3$

h. $-4m^6$

3. Rewrite each expression using exponents.

a. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

b. $a \cdot a \cdot b \cdot b \cdot b$

c. $(-6) \cdot (-6)$

d. $-1 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

4. Evaluate each expression for the given value of x .

a. x^3 , if x is 3

b. x^4 , if x is -2

c. $6x^3$, if x is 3

d. $-5x^4$, if x is -1

B

5. Rewrite each expression using exponents.

a. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

b. $(-3) \cdot (-3) \cdot (-3) \cdot y \cdot y$

c. $\left(-\frac{2}{5}\right) \cdot \left(-\frac{2}{5}\right)$

d. $-13 \cdot m \cdot m \cdot n \cdot n \cdot n$

6. Evaluate the expressions.

a. 7^3

b. 1^{2021}

c. $(0.2)^5$

d. $(-6)^2$

e. -3^4

f. $9 \cdot 2^1$

g. $7 \cdot 1^{10}$

h. $-5 \cdot 3^4$

7. Evaluate each expression for the given value of x .

a. $-\frac{3}{16} \cdot y^2$, if y is 8

b. $\frac{10}{3x^3}$, if x is 5

c. $2ab^2$, if a is 2 and b is 4

d. $-4x^3y^2$, if x is -1 and y is 3

C

8. Does $5^3 = 3^5$? Explain your answer.
9. Does $(-7)^4 = -7^4$? Explain your reasoning.
10. Are $2a^6$ and $(2a)^6$ equivalent? Why or why not?
11. Evaluate $(5 + 4)^2$ and $5^2 + 4^2$. Are they equivalent? Why or why not?
12. Arrange the following powers from least to greatest value: 1^{50} , 4^5 , 5^4 , 2^7 and 7^2 .



1.2

PROPERTIES OF THE NATURAL EXPONENTS



You will:

- know the properties of natural exponents;
- apply the properties of natural exponents;



Discussion

What if there were more terms with the same base? Discuss whether the following examples can be simplified or not. Why so or why not?

a. $8^3 \cdot 8^2 \cdot 8^5 \cdot 8^4$

b. $2^{13} \cdot 2^{12} \cdot 2^7 \cdot 2^6 \cdot 2^{20}$



Discussion

Can the following example be simplified using the multiplication property?

$$3^8 \cdot 5^{12}$$

Why so or why not?



Common mistakes

The Multiplication Property can ONLY be simplified when the bases are SAME.

$$4^2 \cdot 2^3 \neq 8^{2+3}$$

$$4^2 \cdot 4^3 = 4^{2+3} = 4^5$$

There are five properties of the exponent. They are easy to memorize and very helpful in solving problems. The five exponent properties are:

1. Multiplication Property;
2. Division Property;
3. Power of a Power Property;
4. Power of a Product Property;
5. Power of a Quotient Property.

MULTIPLICATION PROPERTY

Is there a rule to help us multiply exponential expressions?

Let's try to complete **Table 1.2.1** by rewriting each of the products as a single power of the base using what we already know:

Product	Repeated Multiplication Form	Exponential Form
$2^3 \cdot 2^5$		
$6^2 \cdot 6^3$		
$10^1 \cdot 10^4$		
$(-4)^2 \cdot (-4)^3$		

Table 1.2.1

a. Do you notice a pattern? Describe the pattern in **Table 1.2.1**. Then write a general rule for multiplying two powers that have the same base.

$$a^n \cdot a^m = \dots$$

b. Let's use your rule to simplify the products in the first column of **Table 1.2.1**. Does your rule give the same results as the third column?

As we can see, to multiply two exponential expressions with the same base, we keep the base and add the exponents. We call this simplifying the exponential expression.

Property 1

Let a be any number and let m and n be natural numbers. Then,

$$a^n \cdot a^m = a^{n+m}$$

Example 1

Use the Multiplication Property of Exponents to simplify.

a. $5^3 \cdot 5^6$

b. $a \cdot a^8$

c. $x^2 \cdot y^4$

Solution:

a. $5^3 \cdot 5^6 = 5^{3+6} = 5^9$



a. Do you notice a pattern? Describe the pattern in Table 1.2.2. Then write a general rule for dividing two powers that have the same base.

$$\frac{a^n}{a^m} = \dots$$

b. Let's use your rule to simplify the quotients in the first column of Table 1.2.2. Does your rule give the same results as the third column?

As we can see, to divide one exponential expression by another with a common base, keep the base the same and subtract exponents.

Property 2

If m and n any natural numbers and $a \neq 0$, then

$$\frac{a^n}{a^m} = a^{n-m}$$

Example 3

Use the Division Property to simplify.

a. $\frac{2^5}{2^4}$

b. $\frac{x^3}{x}$

c. $\frac{3a^2b^5}{ab}$

Solution:

a. $\frac{2^5}{2^4} = 2^{5-4} = 2^1 = 2$

b. $\frac{x^3}{x} = \frac{x^3}{x^1} = x^{3-1} = x^2$

c. Begin by grouping common bases.

$$\frac{3a^2b^5}{ab} = 3 \cdot \frac{a^2}{a^1} \cdot \frac{b^5}{b^1} = 3 \cdot a^{2-1} \cdot b^{5-1} = 3a^1b^4 = 3ab^4$$



Keep in mind

Don't forget that if no exponent is written, it is assumed to be 1. For example,

$$3^1 = 3$$

Similar questions

Page 26-27 questions 2, 6 and 9.

Use the Division Property to simplify.

a. $\frac{6^{18}}{6^{15}}$

b. $\frac{4m^2n^4}{n^3m}$

c. $\frac{a^{2n+3}}{a^{2n-2}}$

Example 4

According to studies, there are many different types of bacteria on phones. The mobile phone is carrying over 2^{50} bacteria. How many bacteria mobile phone has over per cm square if the total area of the mobile phone is 2^8 cm^2 .

Solution:

Use the model to solve the problem.



$$\text{Number of bacteria per cm}^2 = \frac{\text{Total number of bacteria}}{\text{Total area of the mobile phone}}$$

$$\text{Number of bacteria per cm}^2 = \frac{2^{50}}{2^8} = 2^{50-8} = 2^{42}$$

The mobile phone is carrying over 2^{42} bacteria per square cm.

Practice 4

The memory capacities and prices of five MP3 players are shown in the table.

a. How many times more memory does MP3 Player E have than MP3 Player A?

b. How many times more memory does MP3 Player D have than MP3 Player D?

MP3 Player	Memory (GB)	Price
A	2^1	12000 tg
B	2^2	18000 tg
C	2^3	24000 tg
D	2^4	30000 tg
E	2^5	36000 tg

POWER TO A POWER PROPERTY

What does $(3^2)^2$ mean? We can rewrite $(3^2)^2$ first as $3^2 \cdot 3^2$.

$$3^2 \cdot 3^2 = 3^{2+2} = 3^4 = 81$$

Notice that $(3^2)^2 = 3^{2+2}$, or $3^{2 \cdot 2}$. This rule states that the power of power equals the product of exponents: When you raise a power to another power, keep the base and multiply the exponents.

Property 3

Let a be any number and let m and n be natural numbers. Then,

$$(a^n)^m = a^{n \cdot m}$$

Example 5

Use the Power to a Power Property to simplify.

a. $(2^3)^2$

b. $(a^4)^4$

c. $(-b^5)^3$

Solution:

a. $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$

b. $(a^4)^4 = a^{4 \cdot 4} = a^{16}$

c. $(-b^5)^3 = -b^{5 \cdot 3} = -b^{15}$

Practice 5

Use the Power to a Power Property to simplify.

a. $(4^4)^5$

b. $(1^2)^{100}$

c. $(-2^3)^3$

d. $(x^2)^2 \cdot x^3$

Real-life connection

Most people don't give a second thought to using their cell phone everywhere, from their morning commute to the dinner table to the doctor's office. But research shows that cell phones are far dirtier than most people think.

The typical mobile phone is carrying over 161290 bacteria per cm^2 . This is dirtier than

- a kitchen counter (12000 bacteria per cm^2)
- dog's food fish (13612 bacteria per cm^2)
- a doorknob (55761 bacteria per cm^2).

And of course, the most shocking of the facts: your phone is dirtier than a toilet seat.



Discussion

Discuss with your friends. Is $(8^2)^3$ equal to 8^{2+3} ? Explain your reasoning.

Terminology

Capacity - сыйымдылығы/
вместимость

Denominator - бөлшектің бөлімі/
знаменатель дроби

Doorknob - есік тұтқасы/
дверная ручка

Kitchen counter - ас үй үстелі/
кухонный стол

Numerator - бөлшектің алымы/
числитель дроби

Similar questions

Page 26-27 questions 3, 5 and 10.



EXERCISES 1.2

A

1. Simplify the expressions using the multiplication property.

a. $8^3 \cdot 8^4$ b. $3^3 \cdot 3^6$ c. $(-2)^4 \cdot (-2)^5 \cdot (-2)^2$

d. $(-7)^3 \cdot (-7)^2 \cdot (-7)$ e. $b^2 \cdot b^5$

f. $a^5 \cdot a^8$ g. $10a^2 \cdot a^3$ h. $7x^2 \cdot x \cdot x^3$

2. Evaluate each quotient.

a. $\frac{7^6}{7^4}$

b. $\frac{(-4)^4}{(-4)}$

c. $\frac{a^{16}}{a^4}$

d. $\frac{3x^2y^5}{1.5xy^3}$

3. Simplify the expressions. Write the result as a power.

a. $(3^2)^5$

b. $(x^5)^5$

c. $(-a^2)^7$

d. $((-4)^3)^5$

B

4. Simplify the expression using the multiplication property.

a. $(9a^2) \cdot (4a^3)$

b. $(-6b^3) \cdot (-2b^4)$

c. $(3x^5) \cdot (2x^2) \cdot (x^3)$

d. $\left(\frac{2}{5}y^2\right) \cdot (10y^7) \cdot (-5y^6)$

5. Simplify the expression. Write your answers as a power.

a. $(2^2)^3 - (2^3)^2$

b. $3^4 \cdot 3^3 + (-3^2)^3$

6. Decide whether each expression equals 4^6 .

a. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

b. $4 + 4 + 4 + 4 + 4 + 4$

c. $4^2 \cdot 4^4$

d. $4^4 + 4^2$

e. $\frac{4^8}{4^2}$

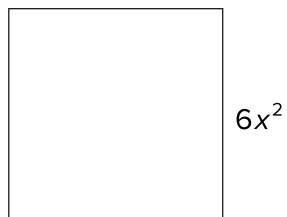
f. $\frac{4^{12}}{4^2}$

g. $(4^2)^3$

h. $(4^3)^3$

C

7. The length of the square is $6x^2$ cm. Find its area.
($S = a^2$)



8. The national postal operator in the Republic of Kazakhstan ('KazPost') delivers about $2^2 \cdot 3 \cdot 5^2 \cdot 13^2$ pieces of parcels each day. There are $2^2 \cdot 3^2 \cdot 5$ days in 6 months. How many pieces of parcels does the KazPost deliver in 6 months? Write your answers as power.



9. In 2020, the total amount of paper and paperboard recycled in Kazakhstan was 200 thousand tons. The population of Kazakhstan in 2020 was about 18 million. On average, how much paper and paperboard did each person recycle? Write your answers as power.



10. Evaluate.

a. $(3^2)^3 - 29$

b. $(4^2)^2 - 240$

c. $\left(\left(\frac{1}{3}\right)^2\right)^2 \cdot 405 - 5$

d. $\left(\left(\frac{2}{5}\right)^2\right)^2 \div \frac{32}{1250}$

